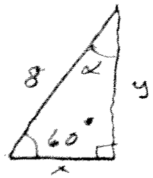
Lösningar
MVE 335, 12 04 10




(1) a) $\alpha+30^\circ+90^\circ=180^\circ \Rightarrow \alpha=\underline{30^\circ}$

b) $x = 8\cos 60^\circ = 8\cdot\frac{1}{2} = \underline{4}$

$y = 8\sin 30^\circ = 8\cdot\frac{\sqrt{3}}{2} = \underline{4\sqrt{3}}$

c) $A = \frac{x\cdot y}{2} = \frac{4\cdot 4\sqrt{3}}{2} = \underline{8\sqrt{3}}$

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(2) a) $x^2+4x+5 = (x+2)^2+1 \Rightarrow$

minsta värdet $=\underline{1}$ (då $x+2=0$, dvs $x=-2$)

b) $(z+2)^2+1=0 \Leftrightarrow (z+2)^2 = -1 \Leftrightarrow$

$\Leftrightarrow z+2=\pm i$  $\underline{z_1=-2+i}$, $\underline{z_2=-2-i}$

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(3) a) $u-2v = (1,-1)-2(1,2) = (-1,-5) \Rightarrow$

$|u-2v| = \sqrt{(-1)^2+(-5)^2} = \underline{\sqrt{26}}$

b) $u-xv = (1,-1)-(x,2x) = (1-x, -1-2x)$

$u-xv \perp u \Leftrightarrow 0 = (u-xv)\cdot(u) = (1-x,-1-2x)\cdot(1,-1) =$

$= 1-x-(-1-2x) = 2+x$  dvs $\underline{x=-2}$

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(4) Cosinussatsen: $6^2=3^2+4^2-2\cdot3\cdot4\cos C$,

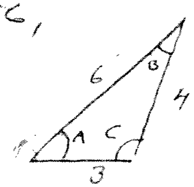
$\cos C = \frac{3^2+4^2-6^2}{2\cdot3\cdot4} = -\frac{11}{24}$

$\underline{C \approx 117{,}3^\circ}$.  Igen:

$4^2 = 3^2+6^2-2\cdot3\cdot6\cos A$

$\cos A = \frac{3^2+6^2-4^2}{2\cdot3\cdot6} = \frac{29}{36}$.  $\underline{A\approx 36{,}3^\circ}$

$B = 180 - A - C = \underline{26{,}4^\circ}$

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(5) a) $\bar{z}_1 - z_2 = 3-4i-(-2+i) = \underline{5-5i}$

b) $|z_1| = \sqrt{3^2+(-4)^2} = \sqrt{9+16} = 5$

$|z_2| = \sqrt{(-2)^2+1^2} = \sqrt{5}$

$|z_1 z_2| = |z_1|\,|z_2| = \underline{5\sqrt{5}}$

$|z_1/z_2| = 5/\sqrt{5} = \underline{\sqrt{5}}$

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(6) a) $4^x - 2^{x+2} = 0 \Leftrightarrow 2^{2x} - 2^2\cdot 2^x$,

$(2^x)^2 - 4\cdot 2^x = 0 = 2^x(2^x-4)=0 \Leftrightarrow 2^x = 4$

$\Leftrightarrow 2^x = 2^2 \Leftrightarrow \underline{x=2}$

b) $\lg(x+2)+\lg(x-2) = 2\lg x - \lg 3$  (obs $x>2$)

$\Leftrightarrow \lg(x+2)(x-2) = \lg\frac{x^2}{3} \Leftrightarrow$

$\Leftrightarrow x^2-4 = \frac{x^2}{3} \Leftrightarrow 2x^2-12=0$, $\underline{x=\sqrt{6}}$.

($-\sqrt{6}$ är falsk rot ty $x>2$)

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(7) Division ger $\frac{2x^2+2x-1}{x^2+x} = 2 - \frac{1}{x^2+x} = 2 - \frac{1}{x(x+1)}$

Ansätt $\frac{1}{x(x+1)} = \frac{a}{x}+\frac{b}{x+1}$.  Mult. med $x(x+1)$

ger $a(x+1)+bx = 1$.  $x=0$ ger $a=1$, $x=-1$ ger $b=-1$

Dvs. $\frac{x^2+2x-1}{x^2+x} = 2 + \frac{1}{x} - \frac{1}{x+1}$

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(8) $\cos^4 x - \sin^4 x = \frac{1}{2} \Leftrightarrow$ (konjugatregeln)

$\Leftrightarrow (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) = \frac{1}{2} \Leftrightarrow$ (Trig.-ettan)

$\Leftrightarrow \cos^2 x - \sin^2 x = \frac{1}{2}$  eller med $\sin^2 x = 1-\cos^2 x$,

$2\cos^2 x - 1 = \frac{1}{2}$, dvs $\cos^2 x = \frac{3}{4}$ och

$\cos x = \pm\frac{\sqrt{3}}{2}$

1) $\cos x = \frac{\sqrt{3}}{2}$ ger $\underline{x = \pm\frac{\pi}{6}+n\cdot 2\pi}$

2) $\cos x = -\frac{\sqrt{3}}{2}$ ger $\underline{x = \pm\frac{5\pi}{6}+n\cdot 2\pi}$