Lösningar MVE 335 12-08-20

1 a) $c^2 = 5^2 + 2^2$ ger $c = \underline{\sqrt{29}\text{ cm}}$

b) $\sin A = \frac{2}{\sqrt{29}}$, $A = \arcsin\left(\frac{2}{\sqrt{29}}\right) \approx \underline{21{,}8^\circ}$

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2 a) $x = 1 \pm \sqrt{1+15} = 1 \pm 4$ $\quad \underline{x_1 = -3,\ x_2 = 5}$

b) $x^2 - 2x - 15 = \underline{(x+3)(x-5)}$

c) Kvadratkomplettera $15 + 2x - x^2 =$

$= -(x^2 - 2x - 15) = -((x-1)^2 - 16) = 16 - (x-1)^2 \le \underline{16}$

med likhet för $x = 1$

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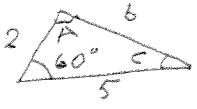
3 Cosinussatsen ger

$b^2 = 2^2 + 5^2 - 2\cdot 2\cdot 5 \cos 60^\circ = 29 - 20\cdot\frac{1}{2} =$

$= 19$, dvs $\underline{b = \sqrt{19}}$



$5^2 = 2^2 + 19 - 2\cdot 2\cdot\sqrt{19}\cos A$, dvs

$\cos A = \frac{2^2+19-5^2}{4\sqrt{19}} = -\frac{1}{2\sqrt{19}}$, $A = 180 - \arccos\left(\frac{1}{2\sqrt{19}}\right) \approx \underline{96{,}6^\circ}$

$C = 180^\circ - 60^\circ - 96{,}6^\circ = \underline{23{,}4^\circ}$

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4 a) $u - 2v = (3,-1) - (4,6) = \underline{(-1,-7)}$

b) $(-1,-7)\cdot(3,-1) = |(-1,-7)|\,|(3,-1)|\cos\theta$

$-3 + 7 = \sqrt{1+49}\sqrt{9+1}\cos\theta$

$\cos\theta = \frac{4}{\sqrt{50}\sqrt{10}} = \frac{2}{5\sqrt{5}}$, $\theta = \arccos\frac{2}{5\sqrt{10}} \approx \underline{79{,}7^\circ}$

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5 a) $(3x^2 - 4)^3 = 8 \iff 3x^2 - 4 = 8^{1/3} = 2 \iff 3x^2 = 6$

$\iff x^2 = 2$, dvs $\underline{x = \pm\sqrt{2}}$

b)

$\lg x = \lg(3x+1) - 1 \iff \lg x = \lg(3x+1) - \lg 10$

$\iff \lg x = \lg\frac{3x+1}{10} \iff x = \frac{3x+1}{10} \iff 10x = 3x+1$

Dvs $7x = 1$ och $\underline{x = 1/7}$

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6 a) $z + z^2 = -2 - i + (-2+i)^2 = -2 - i + 4 - 1 - 4i = \underline{1 - 5i}$

b) $\frac{z}{z^2} = \frac{-2-i}{4-1-4i} = -\frac{(2+i)(3+4i)}{(3-4i)(3+4i)} = -\frac{6-4+3i+8i}{9+16} =$

$= -\frac{2+11i}{25} = \underline{-\frac{2}{25} - \frac{11}{25}i}$

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7 a) $\cos(2x + \frac{\pi}{4}) = 1 \iff 2x + \frac{\pi}{4} = n\cdot 2\pi \iff$

$\iff 2x = n\cdot 2\pi - \frac{\pi}{4} \iff \underline{x = -\frac{\pi}{8} + n\cdot\pi}$

b) $\sin^2 x = \frac{3}{4} \iff \sin x = \pm\frac{\sqrt{3}}{2}$

$\sin x = \frac{\sqrt{3}}{2} \iff \underline{x = \frac{\pi}{3} + n\cdot 2\pi}$ eller $\underline{x = \frac{2\pi}{3} + n\cdot 2\pi}$

$\sin x = -\frac{\sqrt{3}}{2} \iff \underline{x = -\frac{\pi}{3} + n\cdot 2\pi}$ eller $\underline{x = -\frac{2\pi}{3} + n\cdot 2\pi}$

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8 Division ger $\frac{2x^3 + 3x^2 - 3x + 2}{2x^2 + x - 1} = x + 1 - \frac{3x-3}{2x^2+x-1}$

Faktorisera nämnaren: $2x^2 + x - 1 = (2x-1)(x+1)$

Ansätt $\frac{3x-3}{(2x-1)(x+1)} = \frac{a}{2x-1} + \frac{b}{x+1}$. Multiplicera med nämnaren $a(x+1) + b(2x-1) = 3x - 3$

$x = -1$ ger $-3b = -6$, $b = 2$.

$x = \frac{1}{2}$ ger $\frac{3}{2}a = -\frac{3}{2}$, $a = -1$. Alltså är

$\frac{2x^3 + 3x^2 - 3x + 2}{2x^2 + x - 1} = \underline{x + 1 + \frac{1}{2x-1} - \frac{2}{x+1}}$