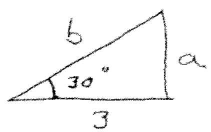
Lösningar till MVE 335 130402

① a) $b\cos 30° = 3 \Rightarrow b = \frac{3}{\cos 30°} = \frac{3}{(\sqrt{3}/2)} = \underline{\underline{2\sqrt{3}}}$

b) $a = b\sin 30° = 2\sqrt{3}\cdot\frac{1}{2} = \underline{\underline{\sqrt{3}}}$

c) $A = \frac{B\cdot h}{2} = \underline{\underline{\frac{3\sqrt{3}}{2}}}$



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② a) $2x^2+x-1=0 \Leftrightarrow x^2+\frac{1}{2}x-\frac{1}{2}=0 \Leftrightarrow$

$\Leftrightarrow x = -\frac{1}{4}\pm\sqrt{(-\frac{1}{4})^2-(-\frac{1}{2})} = -\frac{1}{4}\pm\sqrt{\frac{1}{16}+\frac{8}{16}} = \frac{-1\pm 3}{4}$

$\underline{x_1=-1 \quad x_2=\frac{1}{2}}$

b) $2x^2+x-1 = 2(x^2+\frac{1}{2}x-\frac{1}{2}) = 2((x+\frac{1}{4})^2-\frac{1}{16}-\frac{1}{2}) =$

$= 2((x+\frac{1}{4})^2-\frac{9}{16}) = 2(x+\frac{1}{4})^2-\frac{9}{8}$

Minsta värdet $= \underline{\underline{-\frac{9}{8}}}$ för $x=-\frac{1}{4}$.

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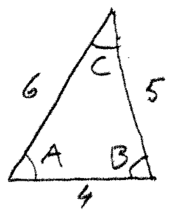
③ Cosinussatsen ger $5^2 = 4^2+6^2-2\cdot4\cdot6\cos A =$

$= 52-48\cos A \Rightarrow \cos A = \frac{52-25}{48} = \frac{27}{48} = \frac{9}{16}$

$\Rightarrow \underline{A} = \arccos\frac{9}{16} \approx \underline{\underline{55{,}8°}}$

Sinussatsen ger $\frac{\sin C}{4} = \frac{\sin A}{5}$,

$\sin C \approx \frac{4\cdot\sin 55{,}8°}{5}$ och $\underline{\underline{C \approx 41{,}4}}$ (C spetsig eftersom $4^2 < 6^2+5^2$). $\underline{\underline{B}} \approx 180°-55{,}8°-41{,}4° \approx \underline{\underline{82{,}8°}}$



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④ $\mathbf{v}-2\mathbf{u} = (1,0,1)-(4,2,0) = (-3,-2,1)$

a) $|\mathbf{v}-2\mathbf{u}| = |(-3,-2,1)| = \sqrt{9+4+1} = \underline{\underline{\sqrt{14}}}$

b) $\mathbf{v}\cdot\mathbf{u} = |\mathbf{v}||\mathbf{u}|\cos\theta$, $\mathbf{v}\cdot\mathbf{u} = 1\cdot2+0\cdot1+1\cdot0 = 2$

$|\mathbf{v}| = \sqrt{1^2+0^2+1^2} = \sqrt{2}$, $|\mathbf{u}| = \sqrt{2^2+1^2+0^2} = \sqrt{5}$

Dvs $\cos\theta = \frac{2}{\sqrt{2}\sqrt{5}} = \frac{\sqrt{2}}{\sqrt{5}}$ och $\underline{\underline{\theta \approx 50{,}8°}}$

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⑤ $\begin{cases} 2x+y-z=0 \\ x+2y+3z=7 \\ -x+y+2z=3 \end{cases} \Leftrightarrow \begin{cases} x+2y+3z=7 \\ 3y+5z=10 \\ -3y-7z=-14 \end{cases} \Leftrightarrow \begin{cases} x+2y+3z=7 \\ 3y+5z=10 \\ -2z=-4 \end{cases}$

3:e ekv ger $\underline{\underline{z=2}}$. 2:a ger $3y+5\cdot2=10$, dvs $\underline{\underline{y=0}}$

1:a ger $x+2\cdot0+3\cdot2=7$, dvs $\underline{\underline{x=1}}$

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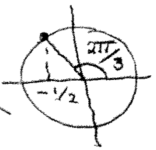
⑥ Eftersom $w\bar{w} = (1+i\sqrt{3})(1-i\sqrt{3}) = 1-3i^2 = 4$ så ger multiplikation med w att $4z = 2w^2 = 2(1+i\sqrt{3})^2 =$
$= 2(1+3i^2+2i\sqrt{3}) = 2(-2+2i\sqrt{3})$ och $\underline{\underline{z=-1+i\sqrt{3}}}$

Bryt ut $|z| = \sqrt{(-1)^2+\sqrt{3}^2} = 2$: $z = 2(-\frac{1}{2}+i\frac{\sqrt{3}}{2}) =$

$= 2(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}) = \underline{\underline{2e^{i\frac{2\pi}{3}}}}$



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⑦ a) $\cos 3x = -\frac{1}{2} \Leftrightarrow 3x = \pm\frac{2\pi}{3}+n\cdot2\pi \Leftrightarrow$

$\underline{\underline{x = \pm\frac{2\pi}{9}+n\cdot\frac{2\pi}{3}}}$, n godtyckligt heltal

b) $\cos 3x = \sin x = \cos(x-\frac{\pi}{2})$. Två fall

1) $3x = x-\frac{\pi}{2}+n\cdot2\pi$, 2) $3x = -(x-\frac{\pi}{2})+n\cdot2\pi$

1) ger $\underline{\underline{x = -\frac{\pi}{4}+n\cdot\pi}}$ 2) ger $\underline{\underline{x = \frac{\pi}{8}+n\cdot\frac{\pi}{2}}}$

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⑧ $R = \sqrt{(2-1)^2+(1-3)^2} = \sqrt{5}$

a) $(x-2)^2+(y-1)^2 = 5$

b) Tangenten är vinkelrät mot linjen genom $(2,1)$ och $(1,3)$. Denna linje har riktningskoefficient $k = \frac{3-1}{1-2} = -2$. Då måste tangenten ha riktningskoefficient $-\frac{1}{k} = \frac{1}{2}$.

Enpunktsformeln ger $y-3 = \frac{1}{2}(x-1)$, dvs tangentens ekvation är $\underline{\underline{x-2y+5=0}}$