1a $1/3$

1b $f(-1) = 0 \quad f'(x) = -2(2x+1)^{-2} \quad f'(-1) = -2$

$y - 0 = -2(x+1) \quad y = -2x - 2$

1c $f' < 0$ om $-2 < x < 3$, dvs avtagende $\Rightarrow f(-1)$ størst.

1d $y - 2.1 = \frac{2.1 - 1.3}{1}(x-3) \quad y = 2.1 + 0.8(-0.3) = 1.86$

2a $f'(x) = 3 - 12x^2 \quad f'(x) = 0 \Rightarrow x = \pm\frac{1}{2}$

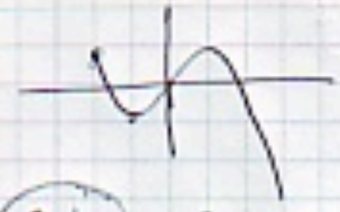
x		$-1/2$		$1/2$	
f'	$-$		$+$		$-$
f	$\searrow$		$\nearrow$		$\searrow$

lok min $\left(-\frac{1}{2}, -1\right)$

lok max $\left(\frac{1}{2}, 1\right)$

størstverdi $f(-1) = 1 \left(= f\left(\tfrac{1}{2}\right)\right)$

minste verdi $= f(2) = -45$

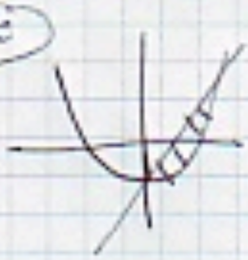


2b $f(-2) = -14 \quad f(-1) = 1 \quad x_0 = -1.5$

$x_1 = x_0 - f(x_0)/f'(x_0) = -1.207$

$x_2 = -1.133 \quad |f(x_2)| = 0.04$

2c



$x^2 - 2 = 4x - 2 \quad x^2 - 4x = 0 \quad x = 0, x = 4$

$$\int_0^4 4x - 2 - (x^2 - 2)\,dx = \int_0^4 (4x - x^2)\,dx$$

$$= \left[2x^2 - \frac{x^3}{3}\right]_0^4 = 32 - \frac{64}{3} = \frac{32}{3}$$

3a $y' = 2be^{bt} + 2at + 1 \quad y'' = 2b^2e^{bt} + 2a$

$$y'' - 4y = \underbrace{2b^2e^{bt} - 8e^{bt}}_{(2b^2-8)e^{bt}} + 2a - 4t - 4at^2$$

$2b^2 - 8 = 0 \quad b = \pm 2 \qquad 2a = 1 \quad a = \frac{1}{2}$

3b $y = -3e^{2t}$

3c $r^2 + 4r + 5 = 0 \quad r = -2 \pm i \quad y = e^{-2t}(C_1 \cos t + C_2 \sin t)$

$y(0) = 2 \Rightarrow \boxed{C_1 = 2} \quad y'(0) = -3 \quad -2e^{-2t}(\quad) + e^{-2t}(-C_1 \sin t + C_2 \cos)$

$-2C_1 + C_2 = -3 \quad C_2 = 2C_1 - 3 = \boxed{1 = C_2}$

(4a) $v = \overrightarrow{P_0P_1} = (-4, -2, 0)$ $\begin{cases} x = -1 - 4t \\ y = 3 - 2t \\ z = 2 \end{cases}$

(4b) $u \cdot v = 7$ $u \cdot w = -7$ $\underline{v \cdot w = 0}$

(4c) $u = \overrightarrow{P_0P_1} = (-5, -1, -1)$ $v = \overrightarrow{P_0P_2} = (2, -4, 2)$

$$u \times v = \begin{vmatrix} e_x & e_y & e_z \\ -5 & -1 & -1 \\ 2 & -4 & 2 \end{vmatrix} = (-6, 8, 22)$$

$\underline{-6x + 8y + 22z} = -12 + 24 + 22 = \underline{34}$

(4d) $\begin{cases} x = 1 + 2t \\ y = 2 \\ z = 3 + 2t \end{cases}$ $x - 6z = 3$

$1 + 2t - 6(3 + 2t) = 3$

$-17 - 10t = 3$ $t = -2$

$\underline{(-3, 2, -1)}$

5a $\displaystyle\int_0^{\pi/2} (1+\sin^2 x)^2 \cos x\,dx = \int_0^1 (1+t^2)^2\,dt = \left[t+\frac{2t^3}{3}+\frac{t^5}{5}\right]_0^1$

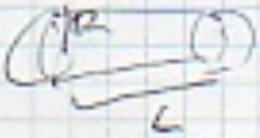
$= 28/15$

5b $\displaystyle\int x(\ln x)^2\,dx = \int e^{2t} t^2\,dt = \frac{1}{2}e^{2t}t^2 - \int \frac{1}{2}e^{2t}\,2t\,dt$

$\displaystyle= \frac{1}{2}e^{2t}t^2 - \frac{e^{2t}}{2}t + \int \frac{e^{2t}}{2}\,dt = \frac{1}{2}e^{2t}\left(t^2 - t + \frac{1}{2}\right) + C$

$\displaystyle= \frac{1}{2}x^2\left((\ln x)^2 - \ln x + \frac{1}{2}\right) + C$

6



$4\pi R^2 + 2\pi R L = 50$

$V = \dfrac{4\pi R^3}{3} + \pi R^2 L$

$L = \dfrac{50 - 4\pi R^2}{2\pi R} \qquad 0 < R \le \sqrt{\dfrac{50}{4\pi}}$

$V = \dfrac{4\pi R^3}{3} + \pi R^2\,\dfrac{50-4\pi R^2}{2\pi R} = 25R - \dfrac{2\pi R^3}{3}$

$V' = \underbrace{25 - 2\pi R^2}_{>0} \qquad V' = 0 \iff R = \sqrt{\dfrac{50}{4\pi}}$

$L = 0$

7 $h' = c\sqrt{h} \qquad \dfrac{h'}{\sqrt{h}} = c \qquad \displaystyle\int \frac{dh}{\sqrt{h}} = \int c\,dt$

$2\sqrt{h} = ct + K$

$2\sqrt{2.5} = c\cdot 0 + K \qquad 2\sqrt{1} = c\cdot 10 + K$

$K = 2\sqrt{2.5} \quad c = \dfrac{2 - 2\sqrt{2.5}}{10}$

$h = 0 \iff ct + K = 0 \iff t = -\dfrac{K}{c}$

$t = \dfrac{-2\sqrt{2.5}}{\dfrac{2(1-\sqrt{2.5})}{10}} = \dfrac{10\sqrt{2.5}}{\sqrt{2.5}-1} = 27 \text{ min}$