SVAR Matematik 2 29/8 - 2014

1a $\frac{2x^3+3}{5x^4-4} \to -3/4,\quad x \to 0$

1b $f'(x) = \frac{4x(x+1)-(2x^2-x)}{(x+1)^2}$ $\quad f'(-2) = \frac{-8\cdot(-1)-(8+2)}{1}$

$f'(-2) = -2$ tangent: $y = -2(x+2) - 10 = -2x - 14$

normal: $y = \frac{1}{2}(x+2) - 10 = \frac{x}{2} - 9$

1c $f'(x) < 0$ om $0 < x < 2$ $\Rightarrow$ $f(0)$ störst

1d $f(3.7) \approx f(4) + (3.7-4)\cdot\frac{f(4)-f(3)}{4-3}$

$= 2.9 - 0.3\cdot 1.2 = 2.54$

2a $f'(x) = 3x^2 - 12x^3 = 3x^2(1-4x) = 0 \iff x=0, x=\frac{1}{4}$

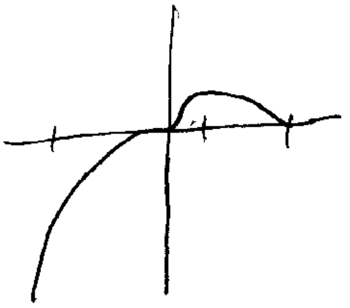
x	-1	0	$1/4$	1
f'	$+$	$+$	$-$	$-$
f	↗	↗	↘	↘

$f(0) = 0$

$f(\frac{1}{4}) = \frac{1}{4^3} - \frac{3}{4^4} = \frac{1}{4^4}$

$f(-1) = -1 - 3 = -4$

$f(1) = 1 - 3 = -2$



lok max $x = -1,\ x = \frac{1}{4}$

min $x = 1$

största värde $= \frac{1}{4^4}$ då $x = \frac{1}{4}$

Minsta —"— $= -4$ då $x = -1$

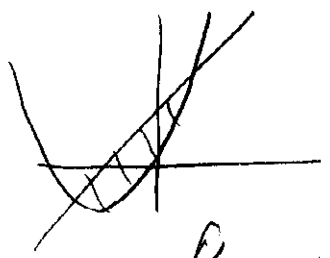
(2b) $f(x) = -2 + x^2 + 2x^3 \quad f'(x) = 2x + 6x^2$

$f(0) = -2 \quad f(1) = 1 \Rightarrow$ rot mellan 0 & 1

t.ex $x_0 = 1 \Rightarrow x_1 = 1 - \frac{1}{8} = 0.875 \quad f(x_1) = 0.105$

$x_2 = 0.858 \quad f(x_2) = 0.002$ / ok

(2c) $3x + 4 = x^2 + 3x \quad x^2 = 4 \quad x = \pm 2$



$$\int_{-2}^{2} (3x + 4 - x^2 - 3x)\,dx =$$

$$\left[4x - \frac{x^3}{3}\right]_{-2}^{2} = 2\cdot\left(8 - \frac{8}{3}\right) = \frac{32}{3}$$

(3a) $y' = 2at + b \quad y'' = 2a$

$2a + 4(2at + b) + 4(at^2 + bt)$

$= \underbrace{4a}_{1}t^2 + (\underbrace{8a + 4b}_{0})t + 2a + 4b$

$a = \frac{1}{4} \quad b = -2a = -\frac{1}{2}$

(3b) $y' = -\frac{4}{3}y \Rightarrow y = Ce^{-\frac{4t}{3}}$

$y(0) = 5 \Rightarrow y = 5e^{-\frac{4t}{3}}$

(alt: kar ekv: $3r + 4 = 0 \Leftrightarrow r = -\frac{4}{3}$

$\Rightarrow y = Ce^{-\frac{4}{3}t}$)

3c

$$r^2 + 6r + 13 = 0 \qquad r = -3 \pm \sqrt{9-13}$$
$$r = -3 \pm 2i$$

$$y = e^{-3t}(A\cos 2t + B\sin 2t)$$

$$y(0) = -2 \iff A = -2$$

$$y'(t) = e^{-3t}(-3A\cos 2t - 3B\sin 2t) + e^{-3t}(-A\sin 2t + 2B\cos 2t)$$

$$y'(0) = -4 \iff -3A + 2B = 0 \qquad B = \frac{3A}{2} = -3$$

$$y(t) = e^{-3t}(-2\cos 2t - 3\sin 2t)$$

4a

$$\mathbf{r} = (2,1,2) + t\big((5,-3,-1) - (2,1,2)\big)$$
$$= (2,1,2) + t(3,-4,-3)$$

$$\begin{cases} x = 2 + 3t \\ y = 1 - 4t \\ z = 2 - 3t \end{cases}$$

4b

$$\begin{vmatrix} e_1 & e_2 & e_3 \\ 3 & 1 & 2 \\ 0 & 3 & 5 \end{vmatrix} = \left(\begin{vmatrix} 1 & 2 \\ 3 & 5 \end{vmatrix}, - \begin{vmatrix} 3 & 2 \\ 0 & 5 \end{vmatrix}, \begin{vmatrix} 3 & 1 \\ 0 & 3 \end{vmatrix} \right)$$
$$= (-1, -15, 9)$$

4c

$$ax+by+cz=d$$

$$3x+y-5z=d$$

$$3\cdot 0+1-5\cdot(-3)=d$$

$$3x+y-5z=16$$

4d

$$\begin{cases} x=5t \\ y=-2+t \\ z=2-2t \end{cases} \quad 20t-3(-2+t)+5(2-2t)=2 \quad 7t=-14 \quad t=-2$$

$$\underline{\underline{(-10,-4,6)}}$$

5a

$$-\frac{\cos 3x}{3}x^2+\int \frac{\cos 3x}{3}2x\,dx$$

$$=\left[-\frac{\cos 3x}{3}x^2+\frac{\sin 3x}{9}2x+\frac{\cos 3x}{27}\cdot 2\right]_0^{\pi/2}$$

$$=-\frac{\pi}{9}-\frac{2}{27}=-\frac{2+3\pi}{27}$$

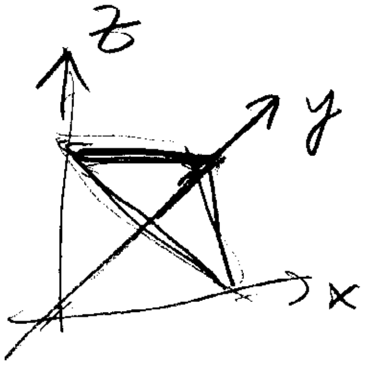
5b

$$\sqrt{x}=t \quad \int \frac{t\cdot 2t}{1+t^2+2t}\,dt=\int \frac{2t^2}{(t+1)^2}\,dt$$

$$=\{t+1=s\}=\int \frac{2(s-1)^2}{s^2}\,ds=\int 2-\frac{4}{s}+\frac{2}{s^2}\,ds$$

$$=2s-4\ln|s|-\frac{2}{s}=4\sqrt{x}-4\ln|1+\sqrt{x}|-\frac{2}{1+\sqrt{x}}+C$$

(6)



plan $ax+by+cz=1$

$$\left.\begin{array}{l} a+2b+3c=1 \\ 2a+2b+c=1 \end{array}\right\} -2$$

$$V=\frac{1}{a}\cdot\frac{1}{b}\cdot\frac{1}{c}\cdot\frac{1}{6}$$

$$\begin{cases} a+2b+3c=1 \\ -2b-5c=-1 \quad (1) \end{cases}$$

$$=\frac{1}{2c}\cdot\frac{1}{1-5c}\cdot\frac{1}{c}\cdot\frac{1}{3}$$

$$\begin{cases} a \quad -2c=0 \\ 2b+5c=1 \end{cases}$$

$$V'=-\frac{6\cdot(2c-15c^2)}{\left(6c^2(1-5c)\right)^2}$$

$$V'=0 \iff c=\frac{2}{15}$$

$$\begin{array}{c|cc} c & \quad 2/15 & \\ \hline V' & - & + \\ & \searrow & \nearrow \end{array}$$

$$b=\frac{1}{2}\left(1-\frac{2}{3}\right)=\frac{1}{6} \qquad a=\frac{4}{15} \qquad \underline{\underline{8x+5y+4z=30}}$$

(7)

$\xrightarrow{}$ 10 l/min $\boxed{200\ l}$ $\xrightarrow{}$ 10 l/min

$x(t)=$ kg salt

$$x'(t)=\frac{50}{1000}\cdot 10-\frac{x(t)}{200}\cdot 10 \qquad x'+\frac{1}{20}x=\frac{1}{2}$$

$$x_h=Ce^{-\frac{t}{20}} \quad x_p=10 \quad x(t)=0.1\,x(0)$$

$$Ce^{-\frac{t}{20}}+10=0.1\cdot(10+C) \qquad Ce^{-\frac{t}{10}}=0.1C-9$$

$$e^{-\frac{t}{10}}=\frac{0.1C-9}{C} \iff t=-10\ln\frac{0.1C-9}{C} \quad (C>90)$$