SVAR Matematik 2 MVE340 14/4-2015

1a) $\dfrac{\cancel{x^3}(5 - \frac{2}{x} + \frac{1}{x^3})}{\cancel{x^3}(\frac{2}{x^2} - 3)} \to -5/3$

1b) $f'(x) = 2x(2x+1)^2 + (x^2-1)\cdot 2(2x+1)\cdot 2 \quad f'(-1) = -2$

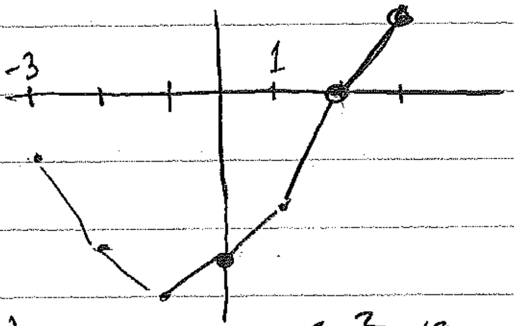
$T:\ y = -2(x+1) \quad N:\ y = \frac{1}{2}(x+1)$

1c)

x		-3		-2		2	
f'	$+$		$-$		$+$		$-$
f	$\nearrow$		$\searrow$		$\nearrow$		$\searrow$

Växande i $-\infty < x \le -3$ resp. $-2 \le x \le 2$

1d)



$f(1.3) \approx f(1) + \dfrac{f(2)-f(1)}{2-1}\cdot(1.3-1)$

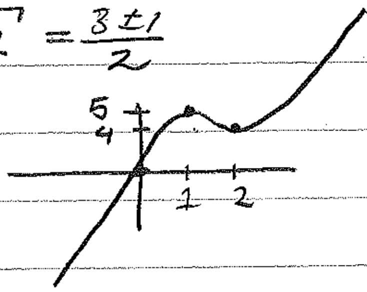
$= -1.7 + 1.7\cdot 0.3 = -1.19$

2a) $f'(x) = 6x^2 - 18x + 12 \quad x = \frac{3}{2} \pm \sqrt{\frac{9}{4} - 2} = \frac{3 \pm 1}{2}$

x		1		2	
f'	$+$		$-$		$+$
f	$\nearrow$		$\searrow$		$\nearrow$

$x = 1$ lok max

$x = 2$ lok min



2b)

x	0	1	2	3
y	1	3	1	-17

$x_0 = 2 \Rightarrow x_1 = x_0 - \dfrac{1 + 4x_0^2 - 2x_0^3}{8x_0 - 6x_0^2} = 2.125 \quad |f(x_1)| = 0.13$

$\underline{x_2} = x_1 - f(x_1)/f'(x_1) \approx \underline{2.11} \quad |f(x_2)| = 0.001$

2c) $x^2 - x = 2x^2 + 3x \iff x^2 + 4x = 0 \quad \int_{-4}^{0}(-x^2 - 4x)\,dx = -\left[\frac{x^3}{3} + 2x^2\right]_{-4}^{0} = \frac{32}{3}$

3a) $y' = 2at + b \quad y'' = 2a \quad 2a + 2at^2 + 2bt = 2at^2 + bt + 2t + 1$

$2a = 1 \iff \underline{a = 1/2} \quad b + 2 = 2b \iff \underline{b = 2}$

3b) $y' = \frac{1}{2}y \quad y = Ce^{\frac{t}{2}} \quad y' = \frac{C}{2}e^{t/2} \quad 3 = \frac{C}{2} \iff C = 6 \quad \underline{y = 6e^{t/2}}$

3c) $r^2 + 6r + 13 = 0 \quad r = -3 \pm \sqrt{9 - 13} = -3 \pm 2i$

$y = e^{-3t}(A\cos 2t + B\sin 2t) \quad y(0) = 2 \iff \underline{2 = A}$

$y' = -3e^{-3t}(A\cos 2t + B\sin 2t) + e^{-3t}(-2A\sin 2t + 2B\cos 2t)$

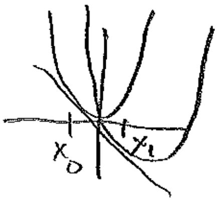
$y'(0) = -3 \iff -3A + 2B = -3 \iff \underline{B = 3/2}$

5a/ $f(x) = \frac{x-3+3}{\sqrt{3-x}} = -\sqrt{3-x} + \frac{3}{\sqrt{3-x}}$

$F(x) = \frac{2}{3}(3-x)\sqrt{3-x} - 6\sqrt{3-x} + C$

b/ $f(x) = \frac{1}{2\left((x+\frac{1}{2})^2+\frac{1}{4}\right)} = \frac{2}{1+(2x+1)^2}$

$F(x) = \arctan(2x+1) + C$

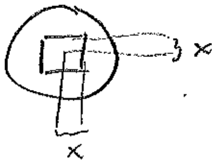
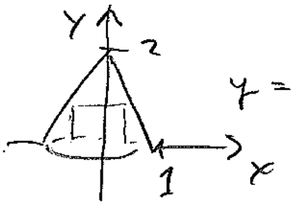
6/ 

lutning $= 2x_0 = 2x_1 - 2 = \frac{x_0^2-(x_1^2-2x_1)}{x_0-x_1}$

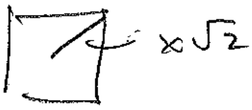
$x_1 = x_0+1$ $\quad 2x_0 = \frac{x_0^2-(x_0+1)^2+2(x_0+1)}{-1} = -1$

$x_0 = -1/2$ $\quad y - x_0^2 = 2x_0\cdot(x-x_0) \iff y-\frac{1}{4} = -(x+\frac{1}{2})$

$y = -x - 1/4$

7/  

$y = 2-2x$



$Vol = (2x)^2\cdot(2-2x\sqrt{2}) = 8x^2(1-x\sqrt{2})$

$V' = 8(2x-3\sqrt{2}x^2) = 8x(2-3\sqrt{2}x)$

$V' = 0 \iff x = \frac{2}{3\sqrt{2}} = \frac{\sqrt{2}}{3}$ $\quad + \; -$ max.

$V_{max} = \left(2\cdot\frac{\sqrt{2}}{3}\right)^2\left(2-2\cdot\frac{\sqrt{2}}{3}\sqrt{2}\right) = \frac{8}{9}\left(2-\frac{4}{3}\right) = \frac{16}{27}\ m^3$

4a) $\overrightarrow{P_1P_2} = (2,-7,-2)$ $\quad \begin{cases} x = 1+2t \\ y = 3-7t \\ z = 0-2t \end{cases}$

4b/ $(2,2,5)\times(0,4,-3) = \begin{vmatrix} e_1 & e_2 & e_3 \\ 2 & 2 & 5 \\ 0 & 4 & -3 \end{vmatrix} = (-26,6,8)$

4c/ $1\cdot x+(-2)\cdot y+4\cdot z = d$ $\quad d = 1\cdot 2-2\cdot 0+4\cdot(-1) = -2$

SVAR: $x-2y+4z = -2$

4d/ $(-2)\begin{cases} 2x-y+3z=-1 \\ 4x-3y+5z=2 \end{cases}$ $\quad \begin{cases} 2x-y+3z=-1 \\ -y-z=4 \end{cases}$ $\quad$ SVAR: $\begin{cases} x = -\frac{5}{2}-2t \\ y = -4-t \\ z = t \end{cases}$