MVE340 150603 (1)

1a) -2

1b) $f'(x) = 2x \cdot 2(x^2-4)(3-x) - (x^2-4)^2$

$f'(1) = 2\cdot(-6)\cdot 2 - 9 = -33$

TANGENT $y = 18 - 33(x-1)$

NORMAL $y = 18 + \frac{x-1}{33}$

1c)

x		-2		-1		1	
f'	$+$		$-$		$+$		$-$
	$\nearrow$		$\searrow$		$\nearrow$		$\searrow$

SVAR: $-2 \leq x \leq -1$ RESP $1 \leq x$

1d) $f(-1.8) \approx 4.5 + \frac{4.5-3}{-1}\cdot 0.2 = 4.2$

2a) $f' = 12x^2 - 6x$

x		0		$1/2$	
f'	$+$		$-$		$+$
f	$\nearrow$		$\searrow$		$\nearrow$

x	y
-1	-7
0	0
$1/2$	$-1/4$
1	1

LOK MIN: $x=-1$, $x = 1/2$

LOK MAX: $x = 0$, $x = 1$

MINSTA VÄRDE $= -7$ STÖRSTA VÄRDE $= 1$

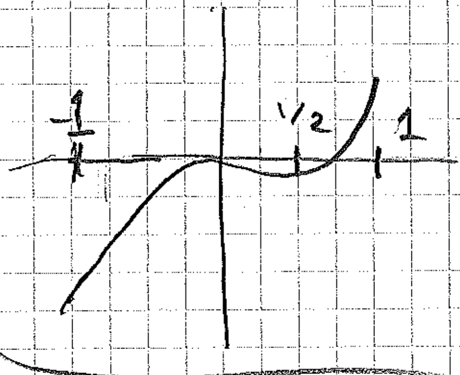
2b) $x_0 = 1$

$x_1 = 1.5$ $f(x_1) = 3.25$

$x_2 = 1.3646$ $f(x_2) = 0.3$

$x_3 = 1.3492$ $f(x_3) = 0.004$

x	y
0	-8
1	-5
2	20

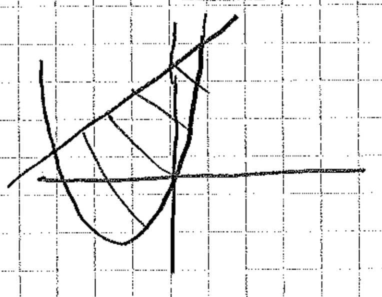


2c) $x^2 + 3x = x + 8$

$x = -1 \pm \sqrt{9} = -1 \pm 3$

$$\int_{-4}^{2} (8 - 2x - x^2)\,dx$$

$$= \left[8x - x^2 - \frac{x^3}{3}\right]_{-4}^{2} = 48 + 14 - \frac{72}{3} = 38$$



2

3a) $y'' = -4a\cos 2t - 4b\sin 2t$

$y'' + 2y = -2a\cos 2t - 2b\sin 2t$

$a = -2,\ b = 1/2$

3b) KAR EKV $3r+1=0$ $r = -1/3$

$y = Ce^{-t/3}$ $y' = -\frac{C}{3}e^{-t/3}$ $y'(0) = -2$

$\Leftrightarrow C = 6$ $\underline{\underline{y = 6e^{-t/3}}}$

3c) KAR EKV $r^2 + 12r + 100 = 0$ $r = -6 \pm 8i$

$y = e^{-6t}(C_1\cos 8t + C_2\sin 8t)$

$y(0) = 4 \Leftrightarrow 4 = C_1\cos 0 + C_2\sin 0 \Leftrightarrow C_1 = 4$

$y' = -6e^{-6t}(\quad) + 8e^{-6t}(-C_1\sin 8t + C_2\cos 8t)$

$y'(0) = 0 \Leftrightarrow 0 = -6C_1 + 8C_2 \Leftrightarrow C_2 = 3$

$y(t) = e^{-6t}(4\cos 8t + 3\sin 8t)$

4a) $\begin{cases} x = t \\ y = -3 - t \\ z = 5 - 2t \end{cases}$ Pktan ligger ej på linjen

4b) $(4,2,1) \times (-1,3,5) = (7,-21,14)$

4c) $2x + 5y - z = 4$

4d) $2(1+2t) - (1-3t) - 2(4t) = 3 \Leftrightarrow 1 - t = 3$

$\Leftrightarrow t = -2$ SVAR: $(-3,7,-8)$

5a) $\sin x = t^2 \quad \cos x\, dx = 2t\, dt$

$$\int 2te^t\, dt = 2te^t - \int 2e^t dt = 2te^t - 2e^t + C$$

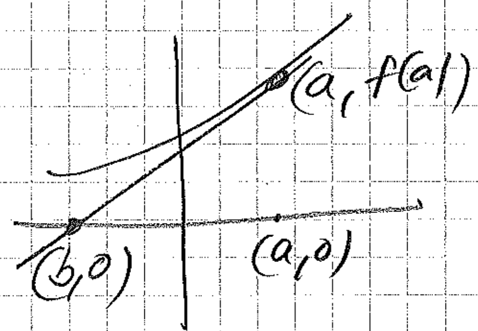
$$= 2(\sqrt{\sin x} - 1)e^{\sqrt{\sin x}} + C$$

5b) $$\int \frac{x}{(x+2)^2+1}dx = \int \frac{t-2}{t^2+1}dt = \int\left(\frac{t}{t^2+1} - \frac{2}{t^2+1}\right)dt$$

$$= \frac{1}{2}\ln(t^2+1) - 2\arctan t + C$$

$$= \frac{1}{2}\ln(x^2+4x+5) - 2\arctan(x+2) + C$$

6)



$$b = a - \frac{f(a)}{f'(a)}$$

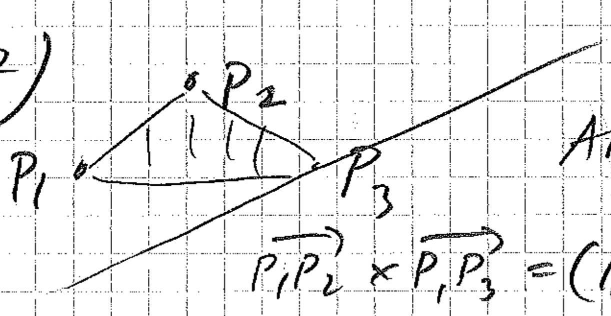
$$\frac{1}{2}\left|f(a)\cdot\frac{f(a)}{f'(a)}\right| = C$$

$$\frac{y'}{y^2} = C_1 \iff \frac{dy}{y^2} = C_1 dx \iff -\frac{1}{y} = C_1x + C_2$$

$$y(0) = 1 \iff -1 = C_2,\ y(1) = 2 \iff -\frac{1}{2} = C_1 - 1$$

$$-\frac{1}{y} = \frac{x}{2} - 1 \iff \underline{\underline{y = \frac{2}{2-x}}}$$

7)



$$\text{Area} = |\overrightarrow{P_1P_2}\times\overrightarrow{P_1P_3}|$$

$$\overrightarrow{P_1P_2}\times\overrightarrow{P_1P_3} = (1,1,-2)\times(2+t, 3-2t, 1-2t)$$

$$= (7-6t, 5, 1-3t)$$

$$\text{Area}^2 = (7-6t)^2 + 25 + (1-3t)^2 = f(t)$$

$$f'(t) = 0 \iff -12(7-6t) - 6(1-3t) = 0$$

$$\iff t = 1 \Rightarrow (x,y,z) = (4,3,2)$$