Sven Matematik 2 NVE320

30/5 – 2013

(a) $$\frac{x^2(-2+\frac{1}{x^2}) \to 0}{x^2(3+\frac{4}{x}+\frac{5}{x^2}) \to 0} \to \frac{-2}{3}, \; x \to -\infty$$

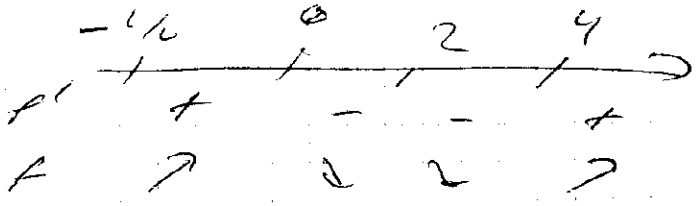
(b) $f(1) = 2 \quad f'(x) = 2(2x-3)\cdot 2 \quad f'(1) = -4$

Tgt $y - 2 = -4(x-1) \quad y = -4x + 6$

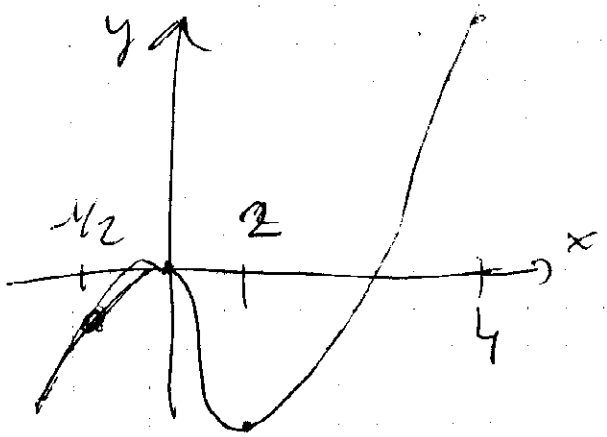
Normal $y - 2 = \frac{1}{4}(x-1) \quad y = \frac{x}{4} + \frac{7}{4}$

(c) $f'(x) = 1\cdot\sqrt{x+3} + x\cdot\frac{1}{2\sqrt{x+3}} \quad f'(1) = 2 + \frac{1}{4} = \frac{9}{4}$

2a) $f'(x) = 6x^2 - 12x^3 = 6x(x-2)$



x	y
-1/2	-7/4
0	0
2	-8
4	32



lok min: $x = 2, -1/2$

lok max: $0, 4$

min värde: $f(2) = -8$

störst värde: $f(4) = 32$

2b)

x	y
0	1
1	-5
2	1

1.934

-2.0598, 1.9343, 1.22

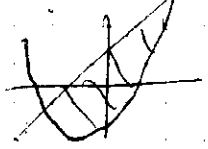
[0,1]

Tg $x_0 = 0.5$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = x_0 - \frac{2x_0^3 - 8x_0 + 1}{6x_0^2 - 8} =$$

$x_1 = 0.078 \quad f(x_1) = 0.39$

$$x_2 = x_1 - \frac{2x_1^3 - 8x_1 + 1}{6x_1^2 - 8} = 0.125 \quad f(x_2) = 0.0013 \text{ OK}$$

2e/ 

$2x+1 = x^2+x-1 \quad x^2-x-2=0$

$x = \frac{1}{2} \pm \sqrt{\frac{1}{4}+2} = \frac{1\pm 3}{2} = -1 \text{ et } 2$

Aire $\int_{-1}^{2}(2x+1-x^2-x+1)dx = \int_{-1}^{2}(-x^2+x+2)dx$

$= \left(-\frac{x^3}{3}+\frac{x^2}{2}+2x\right)_{-1}^{2} = -\frac{8}{3}+2+4-\frac{1}{3}-\frac{1}{2}+2 = \frac{9}{2}$

$\frac{10}{3} - \left(-2+\frac{5}{6}\right) = \frac{10}{3}-\left(\frac{-7}{6}\right) = \frac{27}{6} = \frac{9}{2}$

3a/ $y' = 2t+a \quad y'' = 2$

$y''-2y = 2-2t^2-2at = 2-2t^2+3t$

$\Leftrightarrow -2a = 3 \Leftrightarrow a = -3/2$

3b/ $y'+3y=0 \quad y = Ce^{-3t} \quad y(0)=5 \Rightarrow C=5$

$y = 5e^{-3t}$

3c/ EC. $r^2+4=0 \quad r = \pm 2i$

$y = A\cos 2t + B\sin 2t \Rightarrow y' = -2A\sin 2t + 2B\cos 2t$

$y(0)=3 \Leftrightarrow 3 = A \quad A=3$

$y'(0)=-1 \Leftrightarrow -1 = 2B \quad B=-1/2$

$y = 3\cos 2t - \frac{1}{2}\sin 2t$

4a/ $P = P_0 + tv$ $\quad v = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} - \begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \\ 2 \end{pmatrix}$

$$\begin{cases} x = -1 + 4t \\ y = 5 - 4t \\ z = 2 + 2t \end{cases}$$

4b/ $u \times v = \begin{vmatrix} e_1 & e_2 & e_3 \\ 0 & 1 & -3 \\ 3 & 1 & -2 \end{vmatrix} = e_1 - 9e_2 - 3e_3 = (1, -9, -$

4c/ $\overrightarrow{P_0P} \cdot n = 0 \quad (x-2, y-7, z+1) \cdot (-5, 1, -4) = 0$

$-5(x-2) + y - 7 - 4(z+1) = 0 \quad -5x + y - 4z = -2$

4d/ $\begin{cases} x = 1 + 2t \\ y = -t \\ z = 3 + 4t \end{cases}$ $\quad 2(1+2t) - 6(t) - (3+4t) = 5$

$-6t = 6 \quad t = -1$

$$\begin{cases} x = -1 \\ y = 1 \\ z = -1 \end{cases} \quad (-1, 1, -1)$$

5a/ $t = e^x \quad \frac{dt}{dx} = e^x \quad dt = e^x dx = t\,dx \quad dx = \frac{dt}{t}$

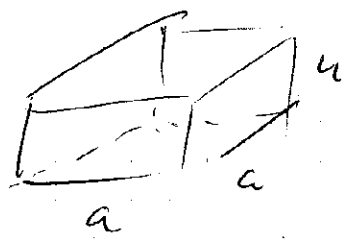
$$\int \frac{t}{t^2 + 2t + 1} \frac{1}{t} dt = \int \frac{1}{(t+1)^2} dt = -\frac{1}{t+1} + C = -\frac{1}{e^x + 1} + C$$

$$\int_0^1 = \left[-\frac{1}{e^x + 1} \right]_0^1 = -\frac{1}{e+1} + \frac{1}{2}$$

5b/ $\int \frac{\ln x}{x^{3/2}} dx = \int x^{-3/2} \ln x \, dx = -2x^{-1/2} \ln x$

$$- \int -2x^{-1/2} \cdot x^{-1} dx = -2x^{-1/2} \ln x + 2 \int x^{-3/2} dx$$

$$= -2x^{-1/2} \ln x - 4x^{-1/2} \cdot \frac{}{} \cdot \int = -\ln 4 - 2 + 4 = 2 - \ln 4$$

6/ 

$a^2 h = 1 \quad h = 1/a^2$

Amin, $A = 4ah + a^2$

$A = a^2 + \frac{4}{a} \quad \frac{dA}{da} = 2a - \frac{4}{a^2} \quad \frac{dA}{da} = 0$

$\Leftrightarrow 2a^3 - 4 = 0 \Leftrightarrow a^3 = 2 \Leftrightarrow a = \sqrt[3]{2}$

$h = \frac{1}{\sqrt[3]{4}} \quad \left(\frac{d^2A}{da^2} = 2 + \frac{8}{a^3} > 0 \Rightarrow \text{lok min} \right)$

7/ $y(t)$ = [illegible] ar [illegible] x(t)-ben t

$y'(t) = -k \cdot y(t) \Rightarrow y(t) = C \cdot e^{-kt}$

$y(0) = M_0$ = [illegible]

$\Rightarrow C = M_0 \Rightarrow y = M_0 e^{-kt}$

$y(1) = 0.7 M_0 \Rightarrow 0.7 M_0 = M_0 e^{-k \cdot 1}$

$0.7 = e^{-k} \quad -k = \ln 0.7 \quad k = -\ln 0.7$

$y(t) = 0.3 M_0 \quad t = ?$

$0.3 M_0 = M_0 e^{-k \cdot t} \quad 0.3 = e^{-kt}$

$\ln 0.3 = -kt \quad t = \frac{\ln 0.3}{-k} = \frac{\ln 0.3}{\ln 0.7}$

≈ 3.38 [illegible].