SVAR MVE340 Matematik del 2 140114

1a

$$\frac{x^2+2+x}{10x^2+5} \to \frac{\cancel{x^2}\left(1+\frac{2}{x^2}+\frac{1}{x}\right)}{\cancel{x^2}\left(10+\frac{5}{x^2}\right)} \to \frac{1}{10},\ x\to\infty$$

1b

$$f'(x) = -(2x+1)^{-3/2} \qquad f(4)=\frac{1}{3} \quad f'(4) = -9^{-3/2} = \frac{-1}{27}$$

$$y-\frac{1}{3} = -\frac{1}{27}(x-4) \qquad y = -\frac{x}{27}+\frac{13}{27}$$

1c

$$f(-3)$$

1d

$$y-(-1.9) = \frac{-1.9-(-0.5)}{2-1}(x-2)$$

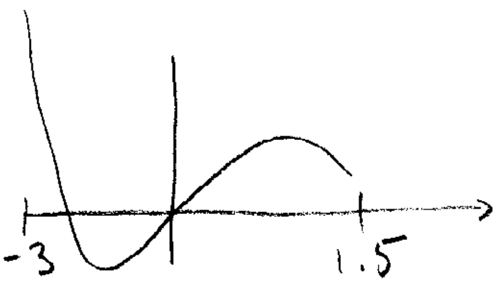
$$y = -1.4(x-2)-1.9 \qquad f(1.7) \approx 1.4\cdot 0.3-1.9 = 1.48$$

2a

$$f'(x) = 9-9x^2 = 0 \iff x=\pm 1$$

		-1		1	
f'	$-$		$+$		$-$

x	-3	-1	1	1.5
$f(x)$	54	-6	6	3.375



Storsta varde : $f(-3)=54$
Minsta —"— : $f(-1)=-6$
lok max : $(-3,54)$ $(1,6)$
lok min $(-1,-6)$ $(1.5,\ 3.375)$

2b

x	-3	-2	-1	
$f(x)$		-3	6	

$$x_0=-2 \quad x_1=-1.786 \quad x_2=-1.763$$

$$|f(x_2)| = 0.003 < 0.05$$

(2c) $x^2+2 = 2x^2+x \Leftrightarrow x^2+x-2=0$

$x = -\frac{1}{2} \pm \sqrt{\frac{1}{4}+2} = \frac{-1\pm 3}{2} \quad x_1=-2 \quad x_2=1$

$$\int_{-2}^{1} x^2+2-(2x^2+x) = \left[2x-\frac{x^2}{2}-\frac{x^3}{3}\right]_{-2}^{1} = \frac{9}{2}$$

(3a) $y' = 2t+a \quad y''=2$

$2+4(2t+a)+4(t^2+at+b) = 4t^2+\overbrace{(8+4a)}^{2}t \underbrace{+2+4a+4b}_{3}$

$a=-\frac{3}{2} \quad b=\frac{7}{4}$

(3b) $2y'+3y=0 \quad y'=-\frac{3}{2}y \quad y=Ce^{-\frac{3}{2}t}$

$y(0)=4 \Rightarrow y=4e^{-\frac{3}{2}t}$

(3c) $8r^2+4r+5=0 \quad r^2+\frac{1}{2}r+\frac{5}{8}=0$

$r = -\frac{1}{4} \pm \sqrt{\frac{1}{16}-\frac{5}{8}} = -\frac{1}{4} \pm i\frac{3}{4}$

$y = e^{-\frac{t}{4}}\left(c_1 \cos\frac{3t}{4} + c_2 \sin\frac{3t}{4}\right)$

$y(0)=2 \Leftrightarrow c_1=2$

$y' = -\frac{1}{4}e^{-\frac{t}{4}}(\quad) + e^{-\frac{t}{4}}\left(-\frac{3}{4}c_1\sin() + \frac{3}{4}c_2\cos()\right)$

$y'(0)=-3 \Leftrightarrow -\frac{1}{2}+\frac{3}{4}c_2 = -3 \Leftrightarrow c_2 = -\frac{10}{3}$

4a) riktnvekt $(4,0,3)-(-2,1,0)=(6,-1,3)$

$$\begin{cases} x = -2+6t \\ y = 1-t \\ z = 3t \end{cases}$$

4b) $\begin{vmatrix} e_x & e_y & e_z \\ 3 & 0 & 2 \\ -1 & 2 & 4 \end{vmatrix} = (-4,-14,6)$

4c) $n=(4,1,-2)$ $\quad P_0=(2,3,1)$

$4x+y-2z+d=0$

$4\cdot 2+3-2\cdot 1+d=0 \quad d=-9$

$4x+y-2z-9=0$

4d) $x=-1+2t,\ y=3-2t,\ z=-3+5t$

$3(-1+2t)+(3-2t)-4(-3+5t)=4$

$\Leftrightarrow 12-16t=4 \Leftrightarrow t=\frac{1}{2}$

$(0,2,-\frac{1}{2})$

5a

$\tan x = t$, $x = \arctan t$, $dx = \frac{1}{1+t^2}dt$

$$\int_0^1 (1+t^2)^2 \frac{dt}{1+t^2} = \left[t + \frac{t^3}{3}\right]_0^1 = \frac{4}{3}$$

5b

$$\int \frac{x}{\sqrt{1+x^2}} \ln x = \sqrt{1+x^2}\ln x - \underbrace{\int \sqrt{1+x^2}\cdot \frac{1}{x}\,dx}$$

$$= \left(\sqrt{1+x^2} = t\right) = \int \frac{t^2}{t^2-1}\,dt = \int 1 + \frac{1}{2}\left(\frac{1}{t-1} - \frac{1}{t+1}\right)$$

$$= \sqrt{1+x^2}\ln x - \sqrt{1+x^2} - \frac{1}{2}\ln\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right)$$

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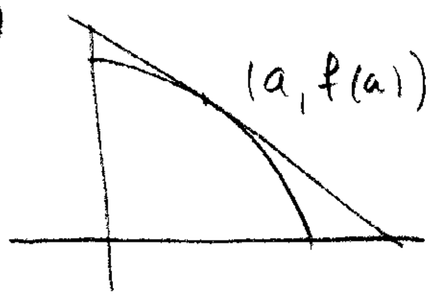

$$y - (3-4a^2) = -8a(x-a)$$

$$y = 0 \Rightarrow x = \frac{3-4a^2}{8a} + a = \frac{3+4a^2}{8a}$$

$$x = 0 \Rightarrow y = 3 - 4a^2 + 8a^2 = 3 + 4a^2$$

$$\text{Area} = \frac{(3+4a^2)^2}{16a} \qquad D = \frac{2(3+4a^2)\cdot 8a\cdot 16a - (3+4a^2)^2\cdot 16}{(16a)^2}$$

$$D = 0 \Leftrightarrow 16a^2 = 3 + 4a^2 \Leftrightarrow 12a^2 = 3 \Leftrightarrow a = (\pm)\frac{1}{2}$$

$(\frac{1}{2}, 2)$

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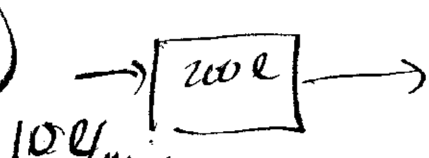

$$y'(t) = -\frac{y(t)}{200}\cdot 10 = -\frac{y(t)}{20}$$

$$y(t) = Ce^{-t/20} \qquad C = y_0$$

$$y(t) = 0.1y_0 \Leftrightarrow e^{-t/20} = 0.1 \Leftrightarrow t = 20\ln 10 \text{ min}$$