Matematik 2 MVE340 140528

**1a**

$$\lim_{x\to\infty} \frac{\cancel{x^3}\left(\frac{1}{x}+\frac{2}{x^3}+3\right)}{\cancel{x^3}\left(\frac{5}{x^2}-4\right)} = \frac{0+0+3}{0-4} = -\frac{3}{4}$$

**1b**

$$f'(x) = \frac{4x-1}{2\sqrt{2x^2-x-1}} \qquad f'(-2) = \frac{-9}{2\cdot 3} = -\frac{3}{2}$$

Tangent: $y = 3 - \frac{3}{2}(x+2) = -\frac{3x}{2}$

Normal: $y = 3 + \frac{2}{3}(x+2) = \frac{2x}{3} + \frac{13}{3}$

**1c**

| $x$ | | $-1.5$ | | $1$ | |
|---|---|---|---|---|---|
| $f'$ | | | $+$ | | |

$f$ växande

$\Rightarrow f(1)$ störst

**1d**

$$f(0.7) \approx f(1) + \frac{f(1)-f(0)}{1-0}\cdot(0.7-1)$$

$$= 1.9 + (1.9-3)\cdot(-0.3)$$

$$= 1.9 + 0.63 = \underline{2.53}$$

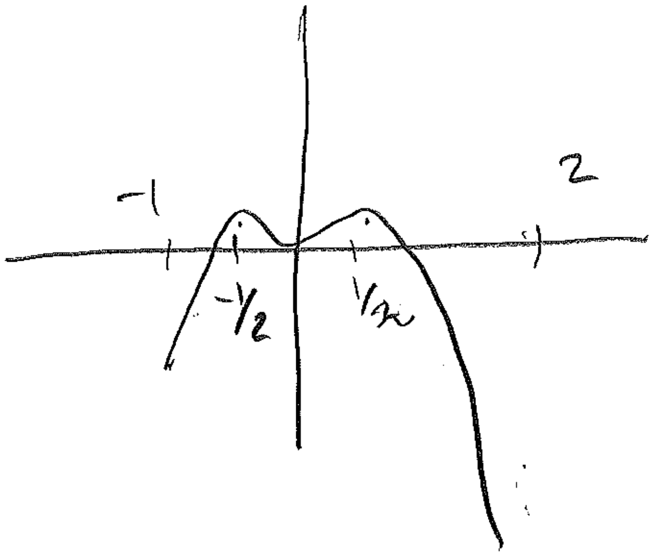
**2a**

$f'(x) = 2x - 8x^3 \quad f'(x) = 0 \Leftrightarrow$

$x = 0$ eller $x = \pm\frac{1}{2}$

| $x$ | | $-1/2$ | | $0$ | | $1/2$ | |
|---|---|---|---|---|---|---|---|
| $f'$ | $+$ | $0$ | $-$ | $0$ | $+$ | $0$ | $-$ |
| $f$ | $\nearrow$ | | $\searrow$ | | $\nearrow$ | | $\searrow$ |

| $x$ | $y = x^2 - 2x^4$ |
|---|---|
| $-1$ | $-1$ |
| $-1/2$ | $1/8$ |
| $0$ | $0$ |
| $1/2$ | $1/8$ |
| $2$ | $-28$ |




lok max $\left(\pm\frac{1}{2}, \frac{1}{8}\right)$

lok min $(-1,-1)$ $(0,0)$ $(2,-28)$

största värde $= \frac{1}{8} = f\left(\pm\frac{1}{2}\right)$

minsta värde $= -28 = f(2)$

2b $f(x) = -9 + 3x^2 + x^3$

| $x$ | $y$ |
|---|---|
| $0$ | $-9$ |
| $1$ | $-5$ |
| $2$ | $11$ |

t.ex $x_0 = 1$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = x_0 - \frac{x_0^3 + 3x_0^2 - 9}{3x_0^2 + 6x_0}$$

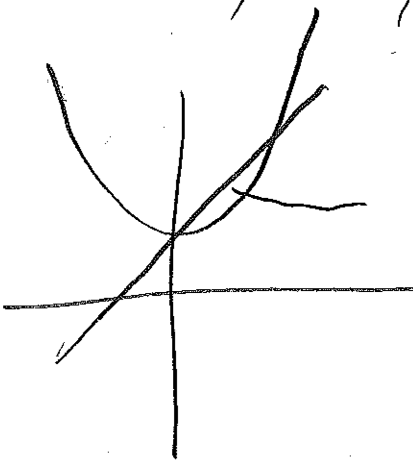
$= 1.556$ $f(x_1) = 2.02 \rightarrow$ fortsätt

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.434$$ $f(x_2) = 0.11 \rightarrow$ fortsätt

$$\underline{x_3} = x_2 - \frac{f(x_2)}{f'(x_2)} = \underline{1.426}$$ $f(x_3) = 0.0004$ stanna

**2c** $3x^2+2 = x+2 \iff 3x^2 = x$

$\iff x_1 = 0,\ x_2 = \frac{1}{3}$



$$\text{Area} = \int_0^{1/3} \left(x+2-(3x^2+2)\right) dx$$

$$= \int_0^{1/3} (x-3x^2)\,dx = \left[\frac{x^2}{2} - x^3\right]_0^{1/3}$$

$$= \frac{1}{18} - \frac{1}{27} = \underline{\underline{\frac{1}{54}}}$$

**3a** $y' = 2at+b \quad y'' = 2a$

$t^2 \cdot 2a = 4at^2 + 4bt + t^2 - t = (4a+1)t^2 + (4b-1)t$

$$\begin{cases} 2a = 4a+1 & a = -1/2 \\ 0 = 4b-1 & b = 1/4 \end{cases}$$

**3b** $2y'+y = 0 \quad y' = -\frac{1}{2}y \Rightarrow y = Ce^{-\frac{1}{2}t}$

$y(0) = 4 \Rightarrow 4 = Ce^0 \iff C = 4 \Rightarrow \underline{y = 4e^{-\frac{1}{2}t}}$

3c) $2r^2+2r+5=0 \quad r=-\frac{1}{2}\pm\sqrt{\frac{1}{4}-\frac{5}{2}}$

$= -\frac{1}{2}\pm i\cdot\frac{3}{2}$

$y = e^{-\frac{1}{2}t}\left(A\cos\frac{3}{2}t + B\sin\frac{3}{2}t\right)$

$y(0) = -2 \Leftrightarrow -2 = \underbrace{e^{0}}_{=1}\left(A\underbrace{\cos 0}_{=1} + B\underbrace{\sin 0}_{=0}\right)$

$\Leftrightarrow A = -2$

$y' = -\frac{1}{2}e^{-\frac{1}{2}t}\left(A\cos\frac{3}{2}t + B\sin\frac{3}{2}t\right) + e^{-\frac{1}{2}t}\cdot\frac{3}{2}\left(-A\sin\frac{3}{2}t\right.$
$\left. + B\cos\frac{3}{2}t\right) \qquad y'(0) = 3$

$\Leftrightarrow 3 = -\frac{1}{2}A + \frac{3}{2}B \Leftrightarrow B = \frac{4}{3}$

$y(t) = e^{-\frac{1}{2}t}\left(-2\cos\frac{3}{2}t + \frac{4}{3}\sin\frac{3}{2}t\right)$

4a) $\overrightarrow{P_0P_1} = (5,-4,3)-(-1,1,2) = (6,-5,1)$

SVAR: $\begin{cases} x = -1+6t \\ y = 1-5t \\ z = 2+t \end{cases} \qquad \left(\overrightarrow{OP} = \overrightarrow{OP_0} + t\,\overrightarrow{P_0P_1}\right)$

(4b) $(-1,1,2)\times(3,-2,4) = \begin{vmatrix} e_1 & e_2 & e_3 \\ -1 & 1 & 2 \\ 3 & -2 & 4 \end{vmatrix}$

$= \left( \begin{vmatrix} 1 & 2 \\ -2 & 4 \end{vmatrix}, -\begin{vmatrix} -1 & 2 \\ 3 & 4 \end{vmatrix}, \begin{vmatrix} -1 & 1 \\ 3 & -2 \end{vmatrix} \right) = \underline{\underline{(8,10,-1)}}$

(4c) $\overrightarrow{P_0P} \cdot n = 0 \Leftrightarrow ax+by+cz=d$

dær $n=(a,b,c)=(4,+1,-2)$

$\Rightarrow 4x+y-2z=d \quad (x,y,z)=(2,3,1)$

$\Rightarrow 4\cdot 2+3-2\cdot 1 = 9 \quad \underline{\underline{4x+y-2z=9}}$

(4d) $\begin{aligned} x &= 5 \\ y &= -1-2t \\ z &= 2+3t \end{aligned} \Rightarrow 4\cdot(5)-5\cdot(2+3t)=7$

$\Leftrightarrow 20-10-15t=7 \Leftrightarrow 3=15t$

$t=\frac{1}{5} \Rightarrow \quad \underline{\underline{(x,y,z)=\left(5,\frac{-7}{5},\frac{13}{5}\right)}}$

**5a)** $\displaystyle\int_0^{\pi/2} \sin x\left(1+\sin^2 x\right)dx = \int_0^{\pi/2} \sin x\left(2-\cos^2 x\right)dx$

$\displaystyle= \left\{ \begin{array}{l} t=\cos x \\ dt=-\sin x\,dx \end{array} \right\} = -\int_1^0 \left(2-t^2\right)dt$

$\displaystyle= \left[2t-\frac{t^3}{3}\right]_0^1 = 2-\frac{1}{3} = \frac{5}{3}.$

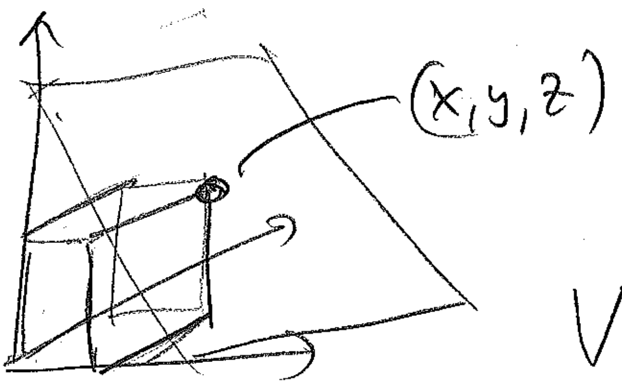
**5b)** $\displaystyle\int \frac{x^5}{\sqrt{1+x^3}}\,dx = \left\{ \begin{array}{l} 1+x^3=t \\ 3x^2dx=dt \end{array} \right\}$

$\displaystyle= \int \frac{(t-1)}{\sqrt{t}}\cdot\frac{dt}{3} = \frac{1}{3}\int\left(\sqrt{t}-\frac{1}{\sqrt{t}}\right)dt$

$\displaystyle= \frac{1}{3}\left(\frac{2}{3}t^{3/2}-2\sqrt{t}\right) = \frac{1}{3}\sqrt{1+x^3}\left(\frac{2}{3}(1+x^3)-2\right) + C$

$\displaystyle= \frac{2}{9}\sqrt{1+x^3}\left(x^3-2\right) + C$

**6)**




$x+2y+3z=4$

$xy=1$

$\text{Volym} = \overbrace{xy}^{1}z$

$\displaystyle= z = \frac{4-x-2y}{3}$

(6) $V = \dfrac{4 - x - \frac{2}{x}}{3}$

$V' = \frac{1}{3}\left(-1 + \frac{2}{x^2}\right)$ $\quad V' = 0 \Leftrightarrow$

$x = (\pm)\sqrt{2}$

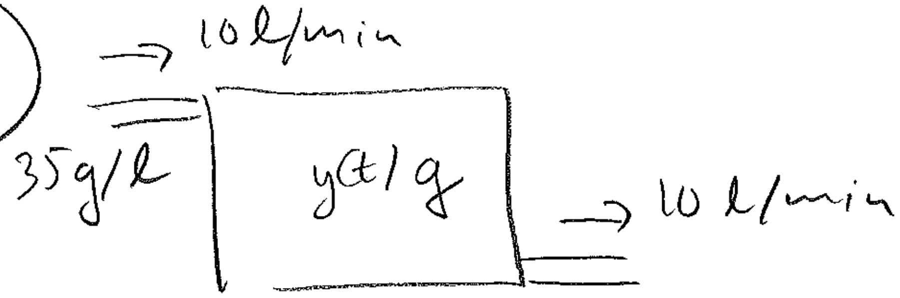
$V'' < 0 \Rightarrow$ max

$x = \sqrt{2} \Rightarrow y = \frac{1}{\sqrt{2}}$ $\quad (x + 2y = 2\sqrt{2} \le 4)$

$Vol = \frac{1}{3}(4 - \sqrt{2} - \sqrt{2}) = \frac{1}{3}(4 - 2\sqrt{2})$

$m^3$

(7)




$y'(t) = \underset{\frac{g}{l}}{35} \cdot \underset{\frac{l}{min}}{10} - \underset{\frac{g}{l}}{\frac{y(t)}{200}} \cdot \underset{\frac{l}{min}}{10} \quad \frac{g}{min}$

$y' = 350 - \frac{y(t)}{20} \quad y' + \frac{y}{20} = 350$

$y_h = Ce^{-\frac{t}{20}} \quad y_p = A \Rightarrow 0 + \frac{A}{20} = 350$

$A = 7000$

(7) $y = y_h + y_p = 7000 + Ce^{-\frac{t}{20}}$

$y(0) = 0 \iff C = -7000$

$y(t) = 20 \cdot 200 = 4000 \Rightarrow t = ?$

$4000 = 7000 - 7000e^{-\frac{t}{20}}$

$\iff$

$e^{-\frac{t}{20}} = \frac{3}{7} \iff -\frac{t}{20} = \ln\frac{3}{7}$

$\iff t = 20\ln\frac{7}{3} = \underline{\underline{17 \text{ min}}}$