Anonym kod	MVE415 Matematisk analys, del 1 170317	Sidnr	Poäng
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1. Till nedanstående uppgifter skall korta lösningar redovisas, samt svar anges, på anvisad plats (endast lösningar och svar på detta blad, och på anvisad plats, beaktas).

(a) Bestäm med hjälp av derivatans definition $f'(x)$ då $f(x) = \frac{x+1}{x+2}$. (2p)

Lösning:

$$\frac{\frac{x+1}{x+2} - \frac{a+1}{a+2}}{x-a} = \frac{(x+1)(a+2) - (a+1)(x+2)}{(x-a)(x+2)(a+2)} =$$

$$= \frac{2x+a-2a-x}{(x-a)(a+2)(x+2)} = \frac{1}{(a+2)(x+2)} \longrightarrow \frac{1}{(a+2)^2}$$

Svar: $f'(a) = 1/(a+2)^2$

(b) Bestäm lokala max/min till funktionen $f(x) = xe^{-x^2}$. (3p)

Lösning:

$$f'(x) = e^{-x^2} + x\cdot(-2x)e^{-x^2} = (1-2x^2)e^{-x^2}$$

$$f'(x) = 0 \iff x = \pm\frac{1}{\sqrt{2}}$$

$-1/\sqrt{2}$ $\quad$ $1/\sqrt{2}$ $\quad x$

t.ex $f'(-1) < 0$ $f'(0) > 0$ $f'(1) < 0$ $\quad - \searrow \quad + \nearrow \quad - \searrow$

Svar: $x = -1/\sqrt{2}$ lok min $\quad x = 1/\sqrt{2}$ lok max

(c) Lös ekvationen $|x-3| + 2 = 4x$. (3p)

Lösning:

$x \geq 3$: $x - 3 + 2 = 4x \iff -1/3 = x \geq 3$ Nej

$x < 3$: $-(x-3) + 2 = 4x \iff 1 = x < 3$ Ja

Svar: $x = 1$

(d) Ange den primitiva funktion till $f(x) = (\sqrt{x}+1)^2$ som uppfyller $F(1) = 2$. (2p)

Lösning:

$$f(x) = x + 2\sqrt{x} + 1$$

$$F(x) = \frac{x^2}{2} + 2x^{3/2}\cdot 2/3 + x + C$$

$$2 = 1/2 + 4/3 + C \qquad C = 1/6$$

Svar:

Var god vänd!

(e) Bestäm inversen till funktionen $y(x) = x^2/(3+2x),\ x \geq 0$. (3p)

Lösning:

$$\frac{x^2}{3+2x} = y \qquad x^2 - 2yx - 3y = 0$$

$$x = y \pm \sqrt{y^2 + 3y} \qquad x \geq 0 \Rightarrow +$$

Svar: $f^{-1}(y) = y + \sqrt{y^2 + 3y}$

(f) Bestäm mha linjär approximation ett närmevärde till $f(3.1)$ om $f(x) = x\sqrt{1+x}$. (3p)

Lösning:

$$f(3.1) \approx f(3) + f'(3) \cdot 0.1$$

$$f'(x) = \sqrt{1+x} + \frac{x}{2\sqrt{1+x}} \qquad f'(3) = 2 + \frac{3}{4}$$

Svar: $f(3.1) \approx 6 + 2.75 \cdot 0.1 = 6.275$

② $f'(x) = \frac{2x(4+2x) - x^2 \cdot 2}{(\quad)^2} = \frac{2x^2+8x}{(\quad)^2} = \frac{2x(x+4)}{(4+2x)^2}$

$f'(x) = 0 \iff x = 0$ eller $x = -4$

$-4 \quad -2 \quad 0$

$+ \nearrow \quad - \searrow \quad - \searrow \quad + \nearrow$

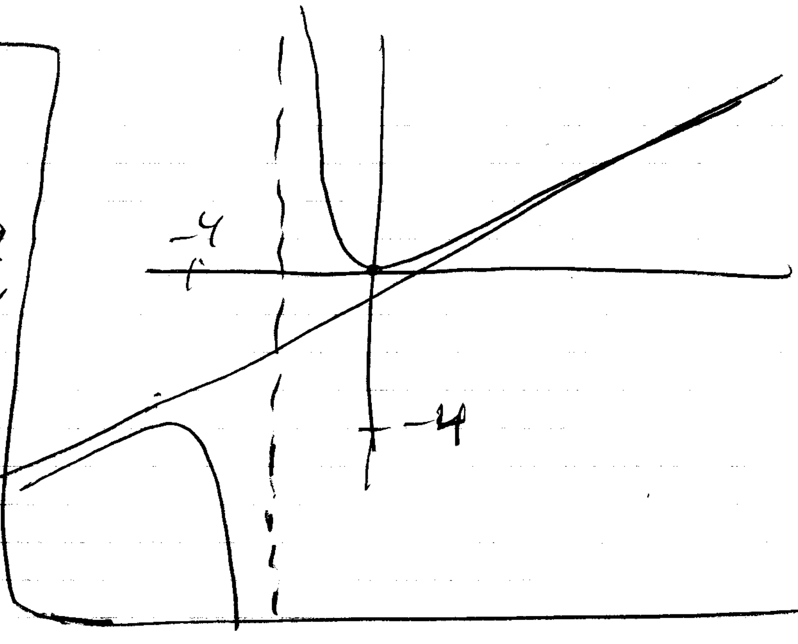
$f(-4) = -4$ lok max

$f(0) = 0$ lok min

$\lim_{x \to -2^\pm} f(x) = \frac{4}{0^\pm} = \pm\infty \quad x = -2$ asymptot

$\frac{f(x)}{x} = \frac{x}{4+2x} \to \frac{1}{2} = k, \qquad f(x) - kx = \frac{x^2}{4+2x} - \frac{x}{2}$

$= \frac{-2x}{4+2x} \to -1 = m \qquad y = \frac{x}{2} - 1$ asymptot



③ $\left(x+\frac{3}{2}\right)^2 +$

$+2\left(y-\frac{3}{2}\right)^2 = -\frac{1}{2}+\frac{9}{4}+\frac{9}{2}$

$\frac{\left(x+\frac{3}{2}\right)^2}{\left(\frac{5}{2}\right)^2} + \frac{\left(y-\frac{3}{2}\right)^2}{\left(\frac{5}{2\sqrt{2}}\right)^2} = 1$

centr $(-3/2, 3/2)$

störst: $\frac{5}{2}$ minst: $\frac{5}{2\sqrt{2}}$ $\qquad y' = -\frac{2x+3}{4y-6}$

$2x+3+4yy'-6y' = 0 \qquad = -\frac{3}{\pm\sqrt{2}}$

④ $\lim_{x\to 0^+} \frac{\sqrt{1-x^2}-1}{x\sqrt{1-x}} \overset{0}{\underset{0}{=}} \lim_{x \to 0^+} \frac{\frac{-x}{\sqrt{1-x^2}}}{\sqrt{1-x} + \frac{-x}{2\sqrt{1-x}}} = \frac{0}{1} = 0$

⑤ $y' = C_1\cos + C_2\sin + 2x(-C_1\sin + C_2\cos)$

$y'' = 2(-C_1\sin + C_2\cos) + 2(-C_1\sin + C_2\cos) + 4x(-C_1\cos - C_2\sin)$

$y''+4y = -4C_1\sin + 4C_2\cos = 3\sin \quad C_1 = -\frac{3}{4} \; C_2 = 0$

(6) $x^2-2x+1=16x^2-8x+1$

$0=15x^2-6x$ $\quad x=0$ eller $x=\frac{2}{5}$

test: $x=0$ Nej $\quad x=\frac{2}{5}$ Ja

(7) $f'(x)=e^{-x}+e^{-2x}-e^{-3x}$

$f(x)=-e^{-x}-\frac{1}{2}e^{-2x}+\frac{1}{3}e^{-3x}+C$

$2=-1-\frac{1}{2}+\frac{1}{3}+C \quad C=\frac{7}{2}-\frac{1}{3}=\frac{19}{6}$

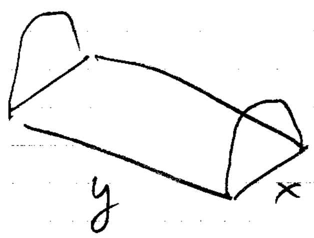
$f'(x)=0 \quad 1+e^{x}-e^{-x}=0 \quad e^{x}=-\frac{1}{2}\pm\sqrt{\frac{1}{4}+1}$

$x=\ln\left(\frac{\sqrt{5}-1}{2}\right) \quad f'(-2)<0 \quad f'(0)>0$ lok min

(8) (i) $\frac{x^{-2}}{x^{-3}(x^2+1)^3}=\frac{d}{dx}\left(\frac{1}{(x^2+1)^2}\cdot\frac{-1}{4}\right)$

(ii) $\frac{2\sin^2x}{\cos^2x}=2\tan^2x=\frac{d}{dx}(2\tan x)-2$

(9)

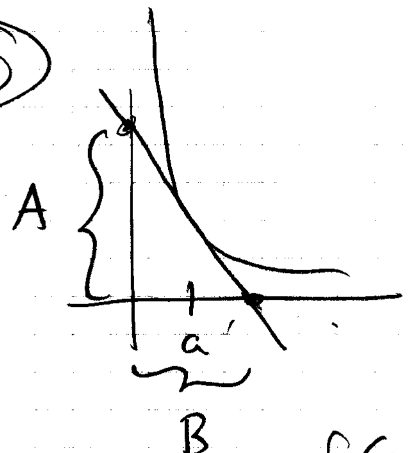


$2y+2x+2\cdot\frac{\pi x}{2}=20$

$Vol=\pi\left(\frac{x}{2}\right)^2\cdot\frac{1}{2}\cdot y$

t.ex. $y=\frac{20-(2+\pi)x}{2} \Rightarrow V(x) \quad V'(x)=0$ ----

(10)



$A=\frac{3}{a^2}+\frac{-6}{a^3}(0-a)=\frac{9}{a^2}$

$B=a-\frac{\frac{3}{a^2}}{\frac{-6}{a^3}}=\frac{3a}{2}$

$f(a)=A^2+B^2 \quad f'(a)=0$ ....