1a) $\int_0^{1/2} 1/\sqrt{1-4x^2}\,dx = \int_0^{1/2} 1/\sqrt{1-(2x)^2}\,dx = [t=2x] = \frac{1}{2}\int_0^1 1/\sqrt{1-t^2}\,dt = \frac{1}{2}[\arcsin t]_0^1$

$= \frac{1}{2}(\arcsin 1 - \arcsin 0) = \frac{1}{2}(\frac{\pi}{2}-0) = \frac{\pi}{4}$ b) $y' = 2y+1 \Leftrightarrow y'-2y=1$

Linjär, IF: $e^{\int -2\,dx} = e^{-2x} \Rightarrow \frac{d}{dx}(e^{-2x}y) = e^{-2x} \Rightarrow e^{-2x}y = \int \frac{d}{dx}(e^{-2x}y)\,dx$

$= \int e^{-2x}\,dx = -\frac{1}{2}e^{-2x} + C \Rightarrow y = -\frac{1}{2} + Ce^{2x}$ e) $\cos(2x) =$

$= \cos(x+x) = \cos x\cos x - \sin x\sin x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 \Rightarrow$

$\int_{\pi/4}^{\pi} \cos^2 x\,dx = \int_{\pi/4}^{\pi} \frac{1+\cos(2x)}{2}\,dx = \frac{1}{2}\left[x + \frac{\sin(2x)}{2}\right]_{\pi/4}^{\pi} = \frac{3\pi}{8} - 1$ c) $\int \cos^3 x\,dx =$

$= \int \cos^2 x\cos x\,dx = \int (1-\sin^2 x)\cos x\,dx = [t=\sin x] = \int 1-t^2\,dt =$

$= t - \frac{t^3}{3} + C = \sin x - \frac{1}{3}\sin^3 x + C$ d) $\int xe^x\,dx = [PI] = xe^x - \int 1\cdot e^x\,dx$

$= xe^x - e^x + C = (x-1)e^x + C$ f) $3y'' - 5y' + 2y = x+1$ Linjär $\Rightarrow$

$y = y_p + y_h$ För y_h: kar. ekv. $0 = 3r^2 - 5r + 2 \Rightarrow r_{1,2} = 1,\ 2/3 \Rightarrow$

$y_h = C_1 e^{(2/3)x} + C_2 e^x$ För y_p: Ansatz $y_p = x^m(ax+b) = (m=0) =$

$= ax+b$ Insättning $\Rightarrow x+1 = 3y_p'' - 5y_p' + 2y_p = 3\cdot 0 - 5a + 2(ax+b) \Rightarrow$

$a = 1/2,\ b = 7/4 \Rightarrow y_p = \frac{1}{2}x + \frac{7}{4}$ och $y = \frac{1}{2}x + \frac{7}{4} + C_1 e^{(2/3)x} + C_2 e^x$

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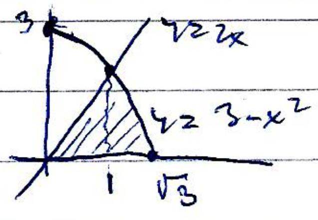
2) Area $= \int_0^1 2x\,dx + \int_1^{\sqrt{3}} 3-x^2\,dx = [x^2]_0^1 + \left[3x - \frac{x^3}{3}\right]_1^{\sqrt{3}}$



$= 1 + \left(3\sqrt{3} - \frac{(\sqrt{3})^3}{3} - (3 - \frac{1}{3})\right) = 2\sqrt{3} - \frac{5}{3}$

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3) a) $\int \frac{1}{\cos^2 x}\,dx = \tan x + C$, c) $\int \frac{1}{e^x+1}\,dx = \left[\begin{matrix} t = e^x+1 \\ dt = e^x dx \Rightarrow dx = \frac{1}{t-1}dt \end{matrix}\right] =$

$= \int \frac{1}{t}\frac{1}{t-1}\,dt = \int \frac{1}{t-1} - \frac{1}{t}\,dt = \ln|t-1| - \ln|t| + C = x - \ln|e^x+1| + C$,

b) $\int_{-1}^{1} e^x - e^{-x}\,dx = \left(\begin{matrix}\text{udda integrand} \\ \text{symmetriskt intervall}\end{matrix}\right) = 0$, d) $\int_0^{\pi/2} \frac{\cos t}{(2+\sin t)^2}\,dt = \left[\begin{matrix} x = 2+\sin t \\ \frac{dx}{dt} = \cos t\,dt \\ 0\to 2,\ \frac{\pi}{2}\to 3\end{matrix}\right] = \int_2^3 \frac{1}{x^2}\,dx =$

$= \left[-\frac{1}{x}\right]_2^3 = -\left(\frac{1}{3} - \frac{1}{2}\right) = 1/6$, e) $\int x\cos x\,dx = [PI] = xe^x - \int 1\cdot e^x\,dx = xe^x - e^x + C = (x-1)e^x + C$

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4) a) $y' - \frac{1}{x}y = x$, Linjär, IF: $e^{\int -\frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x} \Rightarrow \frac{d}{dx}(\frac{1}{x}y) = \frac{1}{x}\cdot x = 1 \Rightarrow$

$\frac{1}{x}y = \int \frac{d}{dx}(\frac{1}{x}y)\,dx = \int 1\,dx = x + C \Rightarrow y = x^2 + Cx$

4b) $y' = y^2$, separabel. Vi ser att $y=0$ lösning och om $y \neq 0$ så har vi $\frac{1}{y^2}\frac{dy}{\cancel{dx}} = 1\,dx \Rightarrow \int \frac{1}{y^2}dy = \int 1\,dx \Rightarrow -\frac{1}{y} = x + C \Rightarrow$
$y = -\frac{1}{x+C} = \frac{1}{C-x}$, $C \in \mathbb{R}$ $\therefore y = \begin{cases} 0 & \text{singulär lösn.} \\ \frac{1}{C-x},\ C\in\mathbb{R}, & \text{allmän lösn.} \end{cases}$
$\therefore$ Sökt lösning är $y \equiv 0 \Leftrightarrow y(x) = 0$, $\forall x$.

5) $y'' + 7y' + 10y = e^x$, linjär. Kar. ekv $0 = r^2 + 7r + 10$
$\Rightarrow r_{1,2} = -2, -5 \Rightarrow y_h = C_1 e^{-5x} + C_2 e^{-2x}$ Ansats:
$y_p = x^m C e^x = (m=0) = Ce^x \Rightarrow y_p = y_p' = y_p'' \Rightarrow 18Ce^x$
$= e^x \Rightarrow C = 1/18 \Rightarrow y_p = \frac{1}{18}e^x \Rightarrow y = y_p + y_h = \frac{1}{18}e^x + C_1 e^{-5x} + C_2 e^{-2x}$

6) Se kurslitteraturen

Överbetygsdelen

7) $\int \sqrt{1-x^2}\,dx = [t = \arcsin x] = \int \cos^2 t\,dt = \int \frac{1+\cos(2t)}{2}dt =$
$= \frac{1}{2}\left(t + \frac{\sin 2t}{2} + C\right) = \frac{1}{2}\arcsin x + \frac{1}{2}x\sqrt{1-x^2} + C$

8) $\int \frac{x^3+4x^2+5x+1}{x^2+4x+5}dx = \int x + \frac{1}{x^2+4x+5}dx = \int x + \frac{1}{(x+2)^2+1}dx =$
$= \frac{x^2}{2} + \arctan(x+2) + C$

9) Se kurslitteraturen; Iterationsformeln för punkten (x_k, y_k) med steglängd h är $y_k = y_{k-1} + h f(x_{k-1}, y_{k-1})$

10) Låt $x(t)$ = mängden salt i behållaren i kg vid tiden t
$\frac{dx}{dt}$ = mängden salt in − mängden salt i kg ut / tidsenhet
tidsenhet = "en hastighet" = 1 min. $\therefore \frac{dx}{dt} = \frac{10\cdot 10}{1000} - \frac{x}{1000}\cdot 10 =$
$= \frac{1}{10} - \frac{x}{100} = \frac{10-x}{100} \Rightarrow x' = \frac{10-x}{100}$ (separabel och linjär) Om
$x \neq 10$ så $\frac{1}{10-x}\frac{dx}{\cancel{dt}} = \frac{1}{100}dt \Rightarrow \int \frac{1}{10-x}dx = \int \frac{1}{100}dt \Rightarrow \ldots \Rightarrow x = 10 + Ce^{-t/100}$, $C \neq 0$
Då $x = 10$ är en lösning fås $x = 10 + Ce^{-t/100}$, $C \in \mathbb{R}$ Vidare ger $x(0) = 50$
att $C = 40$ så $x(40) = 10 + 40e^{-2/5} \approx 36{,}8 \approx 40$ kg salt i behållaren vid $t = 40$