Problem 3.1

areasatsen: $A = \frac{1}{2}\, a b \sin\gamma \quad (*)$

cosinussatsen: $c^2 = a^2 + b^2 - 2ab\cos\gamma \Rightarrow \cos\gamma = \frac{a^2+b^2-c^2}{2ab}$

trig: ettan: $\sin^2\gamma + \cos^2\gamma = 1 \Rightarrow \sin^2\gamma = 1 - \cos^2\gamma$

konjugatregeln: $\sin^2\gamma = (1-\cos\gamma)(1+\cos\gamma) =$

$$= \left(1 - \frac{a^2+b^2-c^2}{2ab}\right)\left(1 + \frac{a^2+b^2-c^2}{2ab}\right)$$

$$= \frac{2ab - a^2 - b^2 + c^2}{2ab} \cdot \frac{2ab + a^2 + b^2 - c^2}{2ab}$$

$$= \frac{c^2 - (a^2 - 2ab + b^2)}{2ab} \cdot \frac{(a^2 + 2ab + b^2) - c^2}{2ab}$$

kvadreringsreglerna: $$= \frac{c^2 - (a-b)^2}{2ab} \cdot \frac{(a+b)^2 - c^2}{2ab}$$

konjugatregeln: $$= \frac{(c+(a-b))(c-(a-b))}{2ab} \cdot \frac{((a+b)+c)((a+b)-c)}{2ab}$$

$$= \frac{(c+a-b)(c+b-a)(a+b+c)(a+b-c)}{(2ab)^2}$$

Således:

$(*)$: $$A = \frac{1}{2}\cancel{a \cdot b} \cdot \sqrt{\frac{(c+a-b)(c+b-a)(a+b+c)(a+b-c)}{(2\cancel{ab})^2}}$$

$$= \frac{1}{4} \cdot \sqrt{(c+a-b)(c+b-a)(a+b+c)(a+b-c)}$$

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Problem 3.2

$$\frac{\sin(u+v)+\sin(u-v)}{2} = \frac{\sin u\cos v + \cancel{\sin v\cos u} + s u\, c v - \cancel{s v\, c u}}{2}$$

$$= \frac{2\, s u\, c v}{2} = s u\, c v \qquad (*)$$

Sätt $\left.\begin{array}{l} u+v=\alpha \\ u-v=\beta \end{array}\right\} \iff \left\{\begin{array}{l} u=\frac{\alpha+\beta}{2} \\ v=\frac{\alpha-\beta}{2} \end{array}\right.$

Då följer det från (*) att: $\frac{\sin\alpha+\sin\beta}{2} = \sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$$\iff \sin\alpha+\sin\beta = 2\cdot\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha-\sin\beta = \sin\alpha+\sin(-\beta) = 2\cdot\sin\frac{\alpha+(-\beta)}{2}\cdot\cos\frac{\alpha-(-\beta)}{2}$$

$$= 2\cdot\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$

$$\frac{\cos(u+v)+\cos(u-v)}{2} = \frac{c u\, c v - \cancel{s u\, s v} + c u\, c v + \cancel{s u\, s v}}{2}$$

$$= \frac{2\, c u\, c v}{2} = c u\, c v$$

$$\left.\begin{array}{l} u+v=\alpha \\ u-v=\beta \end{array}\right\} \Rightarrow \cos\alpha+\cos\beta = 2\cdot\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\frac{\cos(u+v)-\cos(u-v)}{2} = \frac{\cancel{c u\, c v} - s u\, s v - (\cancel{c u\, c v} + s u\, s v)}{2}$$

$$= \frac{-2\, s u\, s v}{2} = -s u\, s v$$

$$\left.\begin{array}{l} u+v=\alpha \\ u-v=\cancel{\alpha}\beta \end{array}\right\} \Rightarrow \cos\alpha-\cos\beta = -2\cdot\sin\frac{\alpha+\beta}{2}\cdot\sin\frac{\alpha-\beta}{2}$$

Problem 3.3(c)

$\boxed{\sin}$ $\quad \sin(x+y) = \sin x + \sin y$

dubbla vinkelns formel

$2\cdot \sin\frac{x+y}{2}\cos\frac{x+y}{2} = 2\cdot \sin\frac{x+y}{2}\cdot \cos\frac{x-y}{2}$ (enligt Problem 3.2)

$\Rightarrow \quad \sin\frac{x+y}{2} = 0 \quad$ eller $\quad \cos\frac{x+y}{2} = \cos\frac{x-y}{2}$

$\frac{x+y}{2} = k\pi \quad k\in\mathbb{Z}$

$\boxed{x+y = 2k\pi}$

$\frac{x+y}{2} = \pm\frac{x-y}{2} + 2k\pi$

$\boxed{x = 2k\pi}$ eller $\boxed{y = 2k\pi}$

$\boxed{\tan}$ $\quad \tan(x+y) = \tan x + \tan y \Leftrightarrow \frac{\tan x+\tan y}{1-\tan x\tan y} = \tan x + \tan y$

$\Leftrightarrow \quad \tan x + \tan y = 0 \quad$ eller $\quad \frac{1}{1-\tan x\tan y} = 1$

$\tan x = -\tan y$

$\tan x = \tan(-y)$

$\boxed{x = -y + k\pi}$ där $k\in\mathbb{Z}$

$1 = 1 - \tan x\tan y$

$0 = \tan x \tan y$

$\tan x = 0$ el. $\tan y = 0$

$\boxed{x = k\pi}$ $\quad$ $\boxed{y = k\pi}$

$\boxed{\cos}$ $\quad$ subst $x = u+v$ o $y = u-v$ $\quad \left(\text{d.v.s. } u = \frac{x+y}{2},\ v = \frac{x-y}{2}\right)$

$\cos(2u) = \cos(u+v) + \cos(u-v)$

$2\cos^2 u - 1 = (\cos u\cos v - \cancel{\sin u\sin v}) + (\cos u\cos v + \cancel{\sin u\sin v})$

$2\cos^2 u - 2\cos u\cos v - 1 = 0$

p,q-formeln ger: $\quad \cos u_{1,2} = \frac{\cos v}{2} \pm\sqrt{\frac{\cos^2 v}{4}+\frac{1}{2}} = \frac{1}{2}\cdot\left(\cos v \pm\sqrt{\cos^2 v + 2}\right)$

Således $\quad u_{1,2,3,4} = \pm\arccos\left(\underbrace{\tfrac{1}{2}\left(\cos v \pm\sqrt{2+\cos^2 v}\right)}\right) + 2k\pi$

då $\quad$ detta uttryck $\in [-1,1]$

$\frac{1}{2}\left(\cos v + \sqrt{2+\cos^2 v}\right) \le 1$ då $\sqrt{2+\cos^2 v} \le 2-\cos v \Leftrightarrow 2+\cos^2 v \le 4 - 4\cos v + \cos^2 v$

$\Leftrightarrow -2 \le -4\cos v \Leftrightarrow \cos v \le \frac{1}{2} \Leftrightarrow \frac{\pi}{3}+2k\pi \le v \le \frac{5}{3}\pi + 2k\pi$

$\frac{1}{2}\left(\cos v - \sqrt{2+\cos^2 v}\right) \ge -1$ då $\sqrt{2+\cos^2 v} \ge 2+\cos v \Leftrightarrow \cos v \ge -\frac{1}{2}$

Problem 3.4

$$\tan 2v = 4\tan v$$

$$\Leftrightarrow \frac{2\tan v}{1-\tan^2 v} = 4\tan v$$

$$\Leftrightarrow 0 = 4\tan v - \frac{2\tan v}{1-\tan^2 v}$$

$$\Leftrightarrow 0 = 2\cdot\tan v\cdot\left(2-\frac{1}{1-\tan^2 v}\right)$$

$$\Leftrightarrow 0 = 2\cdot\tan v\cdot\frac{1-2\tan^2 v}{1-\tan^2 v}$$

$\Leftrightarrow \tan v = 0$ eller $1-2\tan^2 v = 0$

$v = 0 + k\pi$

$\underline{v = k\pi, \text{ där } k\in\mathbb{Z}}$

$1 = 2\tan^2 v$

$\frac{1}{2} = \tan^2 v$

$\pm\sqrt{\frac{1}{2}} = \tan v$

$v = \arctan\left(\pm\frac{1}{\sqrt{2}}\right) + k\pi$

$v = \pm\arctan\frac{1}{\sqrt{2}} + k\pi$

där $k\in\mathbb{Z}$

Problem 3.5/:

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} = \frac{\frac{1}{7} + \frac{3}{4}}{1 - \frac{1}{7}\cdot\frac{3}{4}} = \frac{\frac{4+21}{28}}{1 - \frac{3}{28}} =$$

$$= \frac{\frac{25}{28}}{\frac{25}{28}} = 1$$

$\tan(u+v) = 1 \implies u+v = \arctan 1 + k\pi$

$= \frac{\pi}{4} + k\pi$, dấu $k \in \mathbb{Z}$.

Problem 3.6/

$\gamma = 180° - (\alpha+\beta)$

$$\sin\alpha + \sin\beta + \sin\gamma = s\alpha + s\beta + s(180° - (\alpha+\beta))$$

$$= s\alpha + s\beta + s(\alpha+\beta)$$

$$= s\alpha + s\beta + s\alpha\, c\beta + s\beta\, c\alpha$$

$$= s\alpha(1 + c\beta) + s\beta(1 + c\alpha) = (*)$$

$1 + c\beta = 1 + 2\cdot\cos^2\frac{\beta}{2} - 1 = 2\cdot\cos^2\frac{\beta}{2}$

$1 + c\alpha = \quad \dots \quad = 2\cos^2\frac{\alpha}{2}$

$s\alpha = 2\cdot s\frac{\alpha}{2}\, c\frac{\alpha}{2}$

$s\beta = 2\, s\frac{\beta}{2}\, c\frac{\beta}{2}$

$$(*) = 2\cdot s\frac{\alpha}{2} c\frac{\alpha}{2}\cdot 2c^2\frac{\beta}{2} + 2\cdot s\frac{\beta}{2}\cdot c\frac{\beta}{2}\cdot 2\cdot c^2\frac{\alpha}{2}$$

$$= 4\cdot c\frac{\alpha}{2}\, c\frac{\beta}{2}\cdot\left(s\frac{\alpha}{2} c\frac{\beta}{2} + s\frac{\beta}{2} c\frac{\alpha}{2}\right)$$

$$= 4\cdot c\frac{\alpha}{2}\, c\frac{\beta}{2}\cdot s\frac{\alpha+\beta}{2}$$

$$= 4\cdot c\frac{\alpha}{2}\, c\frac{\beta}{2}\cdot c\left(90° - \frac{\alpha+\beta}{2}\right)$$

$$= 4\cdot c\frac{\alpha}{2}\, c\frac{\beta}{2}\cdot c\left(\frac{180° - (\alpha+\beta)}{2}\right)$$

$$= 4\cdot\cos\frac{\alpha}{2}\cdot\cos\frac{\beta}{2}\cdot\cos\frac{\gamma}{2}$$

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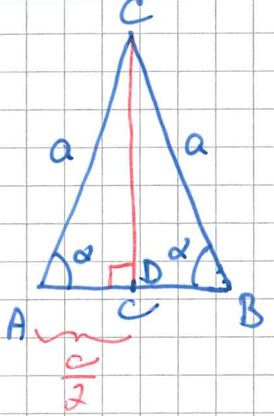
Problem 3.7



(a) ΔADC: $\cos\alpha = \dfrac{|AD|}{|AC|} = \dfrac{c/2}{a}$

$$\Rightarrow \quad a \cdot \cos\alpha = \frac{c}{2}$$

$$\Rightarrow \quad c = 2 \cdot a \cdot \cos\alpha$$

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(b) cosinussetningen i ΔABC:

$$c^2 = a^2 + a^2 - 2 \cdot a \cdot a \cdot \cos\gamma$$
$$= 2a^2 - 2a^2\cos\gamma$$
$$= 2a^2(1-\cos\gamma)$$

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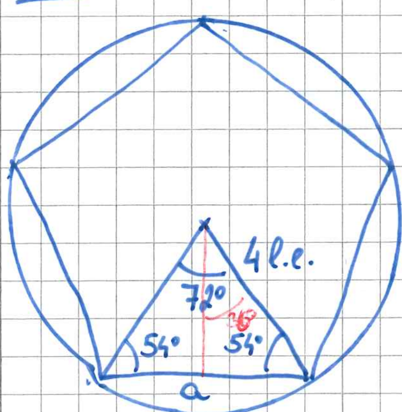
Problem 3.8



(a) $\sin 36° = \frac{a}{2} \div 4$

katet — hypotenusen

$$\Downarrow$$

$$a = 8 \cdot \sin 36° = 8 \cdot \sqrt{\frac{5-\sqrt{5}}{8}}$$
$$= 8 \cdot \sqrt{\frac{10-2\sqrt{5}}{16}} = \frac{8}{4} \cdot \sqrt{10-\sqrt{4 \cdot 5}}$$
$$= 2\sqrt{10-\sqrt{20}} \text{ l.e.}$$

omkretsen: $o = 5a = 10\sqrt{10-\sqrt{20}}$ l.e.

arealet: $A = 5 \cdot \underbrace{\tfrac{1}{2} \cdot 4 \cdot 4 \cdot \sin 72°}_{\text{arealsetningen}} = 40 \cdot \sin 72°$

$$= 10\sqrt{10+\sqrt{20}} \text{ a.e.}$$

(b) $a = 8 \cdot \sin\dfrac{360°}{2k} = 8 \cdot \sin\dfrac{180°}{k}$ l.e.

$o = k \cdot a = 8k \sin\dfrac{180°}{k}$ l.e.

$A = k \cdot \frac{1}{2} \cdot 4 \cdot 4 \cdot \sin\dfrac{360°}{k} = 8k \sin\dfrac{360°}{k}$ a.e.