Uppg. 9.20 e) s. 134

Beräkna $\int \underbrace{\frac{\overbrace{3x+2}^{R(x)}}{x^4-2x^3-8x^2}}_{N(x)} dx$

Lösning:

$N(x) = x^4 - 2x^3 - 8x^2 = x^2(x-4)(x+2)$

Partialbråksuppdelning:

$$\underbrace{\frac{3x+2}{x^2(x-4)(x+2)}}_{N(x)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-4} + \frac{D}{x+2}$$

Gemensam nämnare i HL ger:

$$\frac{3x+2}{N(x)} = \frac{Ax(x-4)(x+2) + B(x-4)(x+2) + Cx^2(x+2) + Dx^2(x-4)}{N(x)} = \frac{(A+C+D)x^3 + (B-2A+2C-4D)x^2 + (-8A-2B)x + (-8B)}{N(x)}$$

Jämför koeff. i täljaren i VL och HL, ger:

$$\begin{cases} A+C+D=0 \\ B-2A+2C-4D=0 \\ -8A-2B=3 \\ -8B=2 \end{cases}$$

Ger: $\boxed{B = -\frac{1}{4}} \Rightarrow -8A = 3 + 2\left(-\frac{1}{4}\right)$

$\Rightarrow \boxed{A = -\frac{5}{16}} \Rightarrow C + D = \frac{5}{16} \Rightarrow D = \frac{5}{16} - C$

$\Rightarrow 2C = 4\left(\frac{5}{16} - C\right) + 2\left(-\frac{5}{16}\right) + \frac{1}{4}$

$\Leftrightarrow 6C = \frac{5}{4} - \frac{5}{8} + \frac{1}{4} = \frac{10 - 5 + 2}{8} = \frac{7}{8}$

$\Rightarrow \boxed{C = \frac{7}{6 \cdot 8}} \Rightarrow D = \frac{5}{2 \cdot 8} - \frac{7}{6 \cdot 8} = \frac{3 \cdot 5 - 7}{6 \cdot 8}$

$= \frac{8}{6 \cdot 8} = \frac{1}{6} \Rightarrow \boxed{D = \frac{1}{6}}$

Alltså: $\frac{3x+2}{x^4 - 2x^3 - 8x^2} = -\frac{5}{16x} - \frac{1}{4x^2} + \frac{7}{48(x-4)} + \frac{1}{6(x+2)}$

vilket ger:

$$\int \frac{3x+2}{x^4 - 2x^3 - 8x^2}\,dx = -\frac{5}{16}\int \frac{1}{x}\,dx - \frac{1}{4}\int \frac{1}{x^2}\,dx + \frac{7}{48}\int \frac{1}{x-4}\,dx$$

$$+\frac{1}{6}\int \frac{1}{x+2}\,dx = -\frac{5\ln|x|}{16} + \frac{1}{4x} + \frac{7\ln|x-4|}{48}$$

$$+ \frac{\ln|x+2|}{6} + C$$

"LÄXA": kontrollera!