Department of
Mathematics
Göteborg

EXAMINATION FOR NUMERICAL LINEAR ALGEBRA, TMA265/MMA600 2010-10-22

DATE: Friday October 22 TIME: 8.30 - 12.30 PLACE: M-building at CTH

Examiner:	Ivar Gustafsson, tel: 772 10 94
Teacher on duty:	Ivar Gustafsson
Solutions:	Will be announced at the end of the exam on the board nearby room MVF21
Result:	Will be sent to you by November 8 at the latest
	Your marked examination can be received at the student's office
	at Mathematics Department, daily 12.30-13
Grades:	To pass requires 13 point, including bonus points from homework assignments
	Grades are evaluated by a formula involving also the computer exercises
Aids:	None (except dictionaries)

Instructions:
- State your methodology carefully. Motivate your statements clearly.
- Only write on one page of the sheet. Do not use a red pen. Do not answer more than one question per page.
- Sort your solutions by the order of the questions. Mark on the cover the questions you have answered. Count the number of sheets you hand in and fill in the number on the cover.

GOOD LUCK!

Question 1.
a) Consider solving a linear system $Ax = b$ with full rank and perturbation δb to the right hand side. Show that the corresponding error δx in the solution is bounded by $\frac{\|\delta x\|}{\|x\|} \leq \kappa(A)\frac{\|\delta b\|}{\|b\|}$, where $\kappa(A)$ is the condition number of A. **(2p)**
b) Present an idea of estimating $\kappa(A)^{-1}$.
Hint: Derive a realistic w in the bound $\frac{1}{\kappa(A)} = \frac{1}{\|A\|\|A^{-1}\|} \leq \frac{\|w\|}{\|A\|\|x\|}$, where $x = A^{-1}w$. **(2p)**

Question 2.
a) Describe matrix by matrix multiplication; C=A*B+C, based on blocking. Derive the ratio q of flops to memory references, when the matrices are n by n and the fast memory contains M words. **(2p)**
b) Mention two operations being on BLAS-3 level. **(1p)**

Question 3.
Consider the (possibly rank-deficient) linear least squares problem

$$(P) \quad \min_x \|Ax - b\|_2.$$

Assume that you have got an SVD-decomposition of A. Show how this can be used to determine the solution to (P) with smallest norm $\|x\|_2$. **(3p)**

Question 4.
Transform the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 2 \\ 1 & 2 & 4 \end{bmatrix}$ to tridiagonal form by similar transformations.
a) Use Householder's reflections. **(2p)**
b) Use Givens' rotations. **(2p)**
c) Use spectrum slicing with shift $\sigma = 1$ to prove that all eigenvalues of A are ≥ 1 and that one eigenvalue is $= 1$. **(2p)**

Question 5.
Let $AX = XB$, where $A \in R^{n\times n}$, $X \in R^{n\times k}$ and $B \in R^{k\times k}$ with $rank(X) = k < n$.
a) Prove that X is a right invariant subspace with respect to A. **(1p)**
b) Prove that the eigenvalues of B are also eigenvalues of A. **(1p)**
c) Explain the concept **deflation** by using a right invariant subspace. **(2p)**

Question 6.
Describe the method: Householder QR-iteration with explicit shift for a nonsymmetric eigenproblem. **(2p)**

Question 7.
Descibe the divide and conquer method for computing eigenvalues and eigenvectors of a symmetric tridiagonal matrix. Do not go into implementation details. **(3p)**