Financial derivative and stochastic analysis, fall 2004

TMA285 or MAM695

Home assignment I

Dead-line November 22, 1.15 pm

1. Exercise 3.6.

2. Suppose you play a sequence of rounds of the game *rock, paper, scissors*. Let the random variables Y_i be defined as

$$Y_i = \begin{cases} 1 & \text{if you win round } i \\ 0 & \text{if there is a draw in round } i \\ -1 & \text{if you loose round } i. \end{cases}$$

   Furthermore, let $X_n = \sum_{i=1}^{n} Y_i$.

   (a) If both you and your opponent randomly (and independently) with equal probability (i.e. $\frac{1}{3}$) choose rock, paper, or scissors each round, compute $E(X_n)$, $V(X_n)$. What is the distribution of X_n?

   (b) What is $E[X_3|\sigma(X_2)]$?

   (c) Let now

$$Y_i' = \begin{cases} 2 & \text{if you win with scissors} \\ 1 & \text{if you win rock or paper} \\ 0 & \text{if there is a draw} \\ -1 & \text{if you loose with rock or paper} \\ -2 & \text{if you loose with scissors.} \end{cases}$$

   Let $Z_n = \sum_{i=1}^{n} Y_i'$. Compute $E(Z_n)$, $V(Z_n)$, illustrate (e.g. plot) the distribution of Z_n.

   (d) Suppose now that your opponent still chooses randomly with equal, but you are free to use any strategy. Analyze a couple of different strategies you may choose.

   (e) Suppose now that your opponent gets to know your strategy, what should he do? Can you pick any strategy that will make you less vulnerable to such information leakage?

3. Let

$$dX = -X^3 \cdot \sin(t)\, dt + dW,$$

   where W is a normal Brownian motion, and where $X(0) = 0$.

   (a) Give (numerical) estimates of the mean, and the variance of $X(t)$ for $0 < t < 0.3$.

   (b) Give an illustration of $Cov(X(t), X(s))$, for $0 < t, s < 0.3$.

   Are your results reasonable?

You may work in pairs. Note that this assignment is not compulsory, but you can gain up to 1.5 bonus points to add to your final score.

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