FINANCIAL DERIVATIVES AND STOCHASTIC ANALYSIS (5p)

(CTH[TMA285]&GU[MAM695])

Period 2, autumn 2006

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Lectures (56 hours): Weeks 44-50: Monday 13^{15} -15 (MVL14), Wednesday 10-12 (MVF21) and 13^{15} -15 (MVF21), and Friday 13^{15} – 15 (MVH11). (Friday November 17 is cancelled; new lecture time November 27, 10- 11^{45} (MVL14))

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Textbook: S. Shreve, Stochastic Calculus for Finance II. Continuous-Time Models, Springer 2004.

CONTENTS

Week 44

Chapters 1 and 2, Probability Theory: Random variable; Expectation; Change of Measure; σ -algebra; Conditional Expectation. **Exercises:** 1.7, 1.8, 1.10, 1.11, 1.14

Week 45

Chapter 3, Brownian Motion: Symmetric random walk; Brownian Motion; Martingale; Quadratic Variation; Markov Property; First Passage Time Distribution. Exercises: 2.3, 2.4, 2.7, 2.8

Week 46

Chapter 4, Stochastic Calculus: Itô integral; Itô-Doeblin formula; Black-Scholes-Merton equation; Option Value; Greeks; Put-Call Parity. Exercises: 3.1, 3.2, 3.4, 3.5, 3.6; Exercise: 4.4

Week 47

Chapter 4, Stochastic Calculus: Multivariable Stochastic Calculus; Gaussian Processes.

Exercises: 4.2, 4.6, 4.7, 4.11

Chapter 5, Risk-Neutral Pricing: Risk-Neutral Measure; Girsanov's Theorem.

Week 48

Chapter 5, Risk-Neutral Pricing: Martingale Representation. Fundamental Theorems of Asset Pricing; Forwards and Futures.

Exercises: 4.13, 4.15, 4.19, 5.2, 5.3, 5.9

Week 49

Chapter 6, Connections with Partial Differential Equations: Feynman-Kac Theorem; Interest Rate Models.

Chapter 7, Exotic Options: Barrier Options. Asian Options. Exercises: 6.1, 6.2, 6.3, 6.6

Week 50

Chapter 9, Change of Numéraire: Foreign and Domestic Risk-Neutral Measures. Forward Measures.

Exercises: 6.7, 6.8, 6.9; Examination September 1, 2006: 1, 2, 5, 6, 7; Examination December 16, 2004: 1, 2, 3, 5

EXAMINATION

Written examinations (4 hours):

December 16, 2006, morning, v

April 13, 2007, morning, v August 28, 2007, morning, v Aid not permitted.

The test comprises 15 points; to pass requires at least 6 points (at GU 11 points or more is graded VG; at Chalmers a result greater than or equal to 9 points and smaller than 12 points is graded 4 and a result greater than or equal to 12 points is graded 5).

Assignments

A number of exercises solved and handed in by the student at the latest Friday, November 24 at 14⁴⁵ will result in a maximum of 1 point at the final examination.

The written examination thus comprises 15 points, where at least 6 points are of theoretic nature. At least 3 points from the theoretic part are collected from the following list:

- (a) Theorem 3.3.4 (b) Theorem 3.4.3 (c) Theorem 3.5.1
- (d) Theorem 3.6.1 (e) Theorem 3.6.2 (f) Theorem 3.7.1
- (g) Theorem 3.7.3. (h) Theorem 4.2.1 (i) Theorem 4.2.2
- (j) Theorem 4.2.3 (k) Theorem 4.4.1 (l) Lemma 4.4.4
- (m) Theorem 4.4.9 (n) Lemma 5.2.1 (o) Lemma 5.2.2
- (p) Theorem 5.2.3 (q) Theorem 5.4.7 (r) Theorem 6.4.1

- (s) Theorem 6.4.3 (t) Theorem 7.2.1 (u) Corollary 7.2.2
- (v) Theorem 9.2.1 (w) Theorem 9.2.2

Göteborg February 12, 2007 Christer Borell