

Partial Differential Equations, Assignment 1A

Consider the two point boundary value problem

$$\begin{aligned} -(au')' + bu' + cu &= f, \quad \text{on } (0, 1) \\ u(1) &= 0, \quad au'(0) = k(u(0) - g). \end{aligned}$$

where a , b , c , f , k and g are given functions.

- a. Give a variational formulation of this problem in a suitable space.
- b. Formulate the corresponding Finite Element Method with a continuous piecewise linear approximation. Write out the elements in the matrices and compute them when a , b , c , and f are constants. Study in particular how the boundary condition is approximated by the FEM.
- c. Prove an *a priori* and an *a posteriori* error estimate under the assumptions that a , $c \geq 0$ and $b = 0$. Formulate an adaptive algorithm based on the *a posteriori* estimate.
- d. Assume $b = 0$ and $c \geq 0$. Formulate the minimization problem which is equivalent to the differential equation above. Show, that they are indeed equivalent.