Mathematics
Chalmers University of Technology
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Partial Differential Equations, Assignment 1B

Consider the two point boundary value problem

$$-(au')' + bu' + cu = f, \quad \text{on} \quad (0,1)$$
$$u(1) = 0, \quad au'(0) = k(u(0) - g).$$

where a, b, c, f, k and g are given functions.

a. Write a finite element code for this problem in e.g. Matlab. It suffices to consider the case $a = 1$, $b = 0$, $c = 1$, $k = 0$, $g = 0$ and $f = 1$.

b. Extend your code to incorporate the term bu' with b a constant.

c. Modify the code to include the Robin boundary condition $au'(0) = k(u-g)$ for arbitrary values of k and g.

A template code in Matlab.

```
function template( )
%
%  This FEM routine solves the equation
%
%          -u''(x) = 1.0, in 0 < x < 1,
%             u(0) = u(1) = 0.
%
%-------------------------------------------------------------------%

clear all % Remove all variables from Matlab's workspace.

x = 0:0.05:1; % Generate node coordinates.

[S, R, b] = assemble(x); % Assemble the matrices and vectors.

A = S + R;

U = A\b; % Solve the linear system of equations.

plot(x,U) % Plot the FEM approximation U(x).

% Subroutines -----------------------------------------------------%

function [S, R, b] = assemble(x)

N = length(x);  % Get the number of nodes.

% Init all matrices and vectors.

b = zeros(N,1); % Load vector
R = zeros(N,N); % Boundary matrix
S = zeros(N,N); % Stiffness matrix

for i = 1:(N-1) % Loop over the sub-intervals and assemble.

  h = x(i+1) - x(i); % Get the mesh size.

  S(i,i)     = S(i,i)     + 1/h;
  S(i,i+1)   = S(i,i+1)   - 1/h;
```

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  S(i+1,i)   = S(i+1,i)   - 1/h;
  S(i+1,i+1) = S(i+1,i+1) + 1/h;

  b(i)   = b(i)   + 0.5 * h;
  b(i+1) = b(i+1) + 0.5 * h;

end

% Set the boundary conditions, u(0) = u(1) = 0.

R(1,1) = 1e4; % A large number here forces u(0) to zero.
R(N,N) = 1e4; % A large number here forces u(1) to zero.
```