Mathematics
Chalmers University of Technology
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Partial Differential Equations, Assignment 1A

Consider the two point boundary value problem

$$-(au')' + bu' + cu = f, \quad \text{on} \quad (0,1)$$
$$u(1) = 0, \quad au'(0) = k(u(0) - g).$$

where a, b, c, f, k and g are given functions.

a. Give a variational formulation of this problem in a suitable space.

b. Formulate the corresponding Finite Element Method with a continuous piecewise linear approximation. Write out the elements in the matrices and compute them when a, b, c, and f are constants. Study in particular how the boundary condition is approximated by the FEM.

c. Prove an *a priori* and an *a posteriori* error estimate under the assumptions that a, $c \geq 0$ and $b = 0$. Formulate an adaptive algorithm based on the *a posteriori* estimate.

d. Assume $b = 0$ and $c \geq 0$. Formulate the minimization problem which is equivalent to the differential equation above. Show, that they are indeed equivalent.