TMA372/MAG800, PARTIAL DIFFERENTIAL EQUATIONS
ASSIGNMENT 1

For each problem you get 1 point
Your bonus point is sum of your assignment points -1 (≤ 2 points)

1. Write down a program that computes the $cG(2)$ finite element approximation of the two-point boundary value problem

$$\begin{cases} -u''(x) = f(x) \quad \text{in} \quad (0,1), \\ u(0) = u(1) = 0. \end{cases} \tag{1}$$

assuming that the user supplies the data vector b (i.e. f is known). Make sure that the code is as efficient as possible using the knowledge from linear algebra and material from chapter 4 of the lecture notes. Test the code for $f(x) = 6x$.

2. Consider the continuous Galerkin $cG(1)$ method for the one-dimensional problem

$$\begin{cases} -\varepsilon u''(x) + u'(x) = 0 \quad \text{in} \quad (0,1), \\ u(0) = 1 \qquad u(1) = 0, \end{cases} \tag{2}$$

(a) Write down the discrete equations for the $cG(1)$ approximation computed on a uniform mesh with M interior nodes.

(b) Compute the approximation for $\varepsilon = 0,01$ and with $M = 10$ and $M = 11$ and compare with the exact (analytic) solution.

(c) Compute the approximation with $M \approx 100$ and compare the result with the exact solution.

3. Consider the initial value problem

$$\begin{cases} \dot{u}(t) + 4u(t) = f(t) \quad \text{for} \quad 0 < t \leq T, \\ u(0) = u_0 \end{cases} \tag{3}$$

(i) Let $u_0 = 1$ and $f(t) = t^2$. Compute the exact solution

(ii) Compute the $cG(1)$ approximation for the solution of the differential equation and determine the condition on the step size that guarantees that U exists.

Hints: For problem 1 you need to read chapter 5. A good starting point for problem 2 might be the Matlab or a C++ code, which solves $-u'' = f$, $u(0) = u(1) = 0$ using $cG(1)$.