

A scheduling problem 2013-02-11 Karin Thörnblad

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Our component specialisation







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Our major engine programmes





Parts processed in the Multitask Cell

~ 8 compressor rear frames for different aero engines and gas turbines. About 30 different jobs are processed in the Multitask Cell.

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The Multitask Production Cell

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Automatic deburring cell Robot deburring

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The routing of a part

Every production order follows a routing in the planning system

One **job** in the multitask cell \leftrightarrow 3-5 route operations

Job processed elsewhere

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The product flow in the multitask cell

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The product flow in the multitask cell

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The queue of parts

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Current detail planning of the multitask cell

Manual planning based on

- Earliest Due Date priority list
- · Other priorities based on the current logistical situation
- The FIFO priority rule (First In First Out) is used in other parts of the factory

Time (h)

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The route operations of the remaining resources are set in a feasible schedule.

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Goal

Produce optimal or near-optimal schedules for the coming shift with ...

... a model that includes enough reality such as e.g. fixture availability and maintenance operations for ...

... real instances comprising about 45 jobs within ...

... a reasonable amount of time, max 15 minutes.

Problem decomposition

Time (h)

The machining problem represented in two ways

Manne family model ("engineer's") (common in textbooks)

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Notation

$k \in \tilde{\mathcal{K}}$, set of resources \tilde{a}_k , first time when resource k is available

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Notation cont'd

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Notation cont'd

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 $j \in \mathcal{J}$, set of jobs

 $\tilde{r}_{j}^{\mathtt{m}}, \text{ release dates}$ $\tilde{d}_{j}, \text{ due dates}$ $\lambda_{jk} = \begin{cases} 1, & \text{if job } j \text{ can be processed on resource } k, \\ 0, & \text{otherwise.} \end{cases}$ $\tilde{p}_{j} \text{ machining processing time of job } j.$

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Notation – a special set of jobs

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 $(j,q) \in \mathcal{Q}$, set of pairs of subsequent jobs for the same physical part,

 \tilde{v}_{jq}^{pm} planned lead time between job j and job q

Time-indexed formulation

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time interval *u* starts at time *ul* hours

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Decision variables

$x_{jku} = \begin{cases} 1, & \text{if job } j \text{ is scheduled on } k \text{ at time } u \\ 0, & \text{otherwise.} \end{cases}$

Binaries:

$$|\mathcal{J}||\mathcal{K}||\mathcal{T}|$$

Test case No 3 (20 jobs): 9300

Time variables

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$$s_j = t_j + \tilde{p}_j^{\text{pm}}$$
 the completion time of job
 $h_j = \max\{s_j - \tilde{d}_j; 0\}$ the tardiness of job j

Time-indexed formulation with nail variables

$$\begin{array}{ll} \text{Minimize} & \sum_{j \in \mathcal{J}} (As_j + Bh_j), \\ \text{subject to} & \sum_{k \in \tilde{\mathcal{K}}} \sum_{u \in \mathcal{T}} x_{jku} = 1, \qquad j \in \mathcal{J}, \\ & \sum_{u \in \mathcal{T}} x_{jku} \leq \lambda_{jk}, \qquad j \in \mathcal{J}, \ k \in \tilde{\mathcal{K}}, \\ & \sum_{u \in \mathcal{T}} \sum_{u \in \mathcal{T}} x_{jku} \leq 1, \qquad k \in \tilde{\mathcal{K}}, u \in \mathcal{T}, \\ & \sum_{j \in \mathcal{J}} \sum_{\nu = (u - \tilde{p}_j + 1)_+}^{u = 1/2} x_{jk\nu} \leq 1, \qquad k \in \tilde{\mathcal{K}}, u \in \mathcal{T}, \\ & \sum_{k \in \tilde{\mathcal{K}}} \left(\sum_{\mu = 0}^{u} x_{jk\mu} - \sum_{\nu = 0}^{u + \tilde{v}_{jq}^{\text{pm}}} x_{qk\nu} \right) \geq 0, \qquad (j,q) \in \mathcal{Q}, \ u = 0, \dots, \mathbb{T} - \tilde{v}_{jq}^{\text{pm}}, \\ & x_{jku} = 0, \qquad (j,q) \in \mathcal{Q}, \ u = \mathbb{T} - \tilde{v}_{jq}^{\text{pm}}, \dots, \mathbb{T}, \\ & \sum_{k \in \tilde{\mathcal{K}}} \sum_{u \in \mathcal{T}} ux_{jku} + \tilde{p}_j^{\text{pm}} = s_j, \qquad j \in \mathcal{J}, \\ & h_j = \max\{s_j - \tilde{d}_j; 0\}, \ j \in \mathcal{J}, \\ & x_{jku} = 0, \qquad j \in \mathcal{J}, \ k \in \tilde{\mathcal{K}}, u = 0, \dots, \max\{\tilde{r}_j^{\text{m}}, \tilde{a}_k\} - 1, \\ & x_{jku} \in \{0,1\}, \qquad j \in \mathcal{J}, \ k \in \tilde{\mathcal{K}}, u \in \mathcal{T}, \end{array}$$

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Time-indexed formulation with nail variables

Objective: Minimize the weighted sum of Minimize $\sum (As_j + Bh_j),$ completion times and tardiness. $j \in \mathcal{J}$ subject to $\sum \sum x_{jku} = 1$, One job is scheduled only once $k \in \tilde{\mathcal{K}} \ u \in \mathcal{T}$ Each job can only be assigned to an $\sum x_{jku} \leq \lambda_{jk},$ allowed resource k $u\!\in\!\mathcal{T}$ $\sum \qquad \sum \qquad x_{jk\nu} \le 1,$ Only one job at a time can be processed on resource k $j \in \mathcal{J} \nu = (u - \tilde{p}_i + 1)_+$ $\left[\sum_{k\in\tilde{\mathcal{K}}} \left(\sum_{\mu=0}^{u} x_{jk\mu} - \sum_{\nu=0}^{u+\tilde{v}_{jq}^{\mathsf{pm}}} x_{qk\nu}\right) \ge 0,\right]$ $(j,q) \in \mathcal{Q}, \ u = 0, \ldots, \mathtt{T} - \tilde{v}_{ja}^{\mathtt{pm}},$ $(j,q) \in \mathcal{Q}, \ u = T - \tilde{v}_{jq}^{pm}, \ldots, T,$ $x_{jku} = 0,$ $\sum \sum u x_{jku} + \tilde{p}_j^{\mathsf{pm}} = s_j,$ $j \in \mathcal{J},$ $k \in \tilde{\mathcal{K}} u \in \mathcal{T}$ $h_j = \max\{s_j - \tilde{d}_j; 0\}, j \in \mathcal{J},$ $j \in \mathcal{J}, k \in \tilde{\mathcal{K}}, u = 0, \dots, \max{\{\tilde{r}_{i}^{\mathtt{m}}, \tilde{a}_{k}\}} - 1,$ $x_{jku} = 0,$ $j \in \mathcal{J}, \ k \in \tilde{\mathcal{K}}, \ u \in \mathcal{T},$ $x_{jku} \in \{0,1\},\$

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A planned lead time v_{jq} has to elapse between jobs on the same part

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Time-indexed formulation with nail variables

 $\sum (As_j + Bh_j),$ Minimize $j \in \mathcal{J}$ subject to $\sum \sum x_{jku} = 1$, $k \in \tilde{\mathcal{K}} \ u \in \mathcal{T}$ $\sum x_{jku} \leq \lambda_{jk},$ $\sum_{n=1}^{u} \sum_{jk\nu \leq 1, jk\nu \leq 1}^{u} x_{jk\nu \leq 1, jk\nu \leq 1}$ $j \in \mathcal{J} \nu = (u - \tilde{p}_i + 1)_+$ $\sum_{k\in\tilde{\mathcal{K}}} \left(\sum_{\mu=0}^{u} x_{jk\mu} - \sum_{\nu=0}^{u+\tilde{v}_{jq}^{\mu}} x_{qk\nu} \right) \ge 0,$ The starting time of job j $x_{jku} = 0$, $\sum_{k \in \tilde{\mathcal{K}}} \sum_{u \in \mathcal{T}} u x_{jku} + \tilde{p}_j^{\text{pm}} = s_j,$ $k \in \tilde{\mathcal{K}} u \in \mathcal{T}$ $h_i = \max\{s_i - \tilde{d}_i; 0\},$ Definition of tardiness $x_{jku} = 0,$

 $x_{jku} \in \{0,1\},\$

Objective: Minimize the weighted sum of completion times and tardiness.

One job is scheduled only once

Each job can only be assigned to an allowed resource k

Only one job at a time can be processed on resource k

Planned lead time between jobs on same part

Definition of completion time

$$j \in \mathcal{J}, k \in \tilde{\mathcal{K}}, u = 0, \dots, \max{\{\tilde{r}_j^{\mathtt{m}}, \tilde{a}_k\}} - 1,$$

Job *j* cannot start in resource *k* before the job's release date or before k is available

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A different formulation of the objective

Same objective formulated without the continuous variables for completion times and tardiness. The computation times needed for solving this model with CPLEX are decreased in comparison with the model on the previous slide.

$$\begin{split} & \min \sum_{j \in \mathcal{J}} \sum_{k \in \tilde{\mathcal{K}}} \sum_{u \in \mathcal{T}} \left(\left(A(u + \tilde{p}_{j}^{\mathsf{pm}}) + B(u + \tilde{p}_{j}^{\mathsf{pm}} - d_{j})_{+} \right) x_{jku} \right), \\ & \text{subject to} \qquad \sum_{k \in \tilde{\mathcal{K}}} \sum_{u \in \mathcal{T}} x_{jku} = 1, \qquad j \in \mathcal{J}, \\ & \sum_{u \in \mathcal{T}} x_{jku} \leq \lambda_{jk}, \quad j \in \mathcal{J}, \quad k \in \tilde{\mathcal{K}}, \\ & \sum_{j \in \mathcal{J}} \sum_{\nu = (u - \tilde{p}_{j} + 1)_{+}}^{u \in \mathcal{T}} x_{jk\nu} \leq 1, \qquad k \in \tilde{\mathcal{K}}, u \in \mathcal{T}, \\ & \sum_{k \in \tilde{\mathcal{K}}} \left(\sum_{\mu = 0}^{u} x_{jk\mu} - \sum_{\nu = 0}^{u \neq \tilde{v}_{jq}^{\mathsf{pm}}} x_{jk\nu} \right) \geq 0, \qquad (j, q) \in \mathcal{Q}, \quad u = 0, \dots, \mathbb{T} - \tilde{v}_{jq}^{\mathsf{pm}}, \\ & \qquad x_{jku} = 0, \qquad (j, q) \in \mathcal{Q}, \quad u = \mathbb{T} - \tilde{v}_{jq}^{\mathsf{pm}}, \dots, \mathbb{T}, \\ & \qquad x_{jku} \in \{0, 1\}, j \in \mathcal{J}, \quad k \in \tilde{\mathcal{K}}, \quad u \in \mathcal{T}. \end{split}$$

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The engineer's model (with continuous time variables)

$$z_{jk} = \begin{cases} 1, & \text{if job } j \text{ is scheduled on } k \\ 0, & \text{otherwise.} \end{cases}$$
$$y_{jqk} = \begin{cases} 1, & \text{if job } j \text{ is processed before job } q \text{ on } k \\ 0, & \text{otherwise.} \end{cases}$$

Binaries:
$$|\mathcal{J}||\mathcal{K}| + |\mathcal{J}||\mathcal{J}||\mathcal{K}|$$

Test case No 3 (20 jobs): 2100

Time variables

$$t_j = \text{starting time}$$

 $s_j = t_j + p_j + p_j^{\text{pm}}$ completion time of job j
 $h_j = \max\{0, s_j - d_j\}$ tardiness of job j

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The engineer's model

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Minimize

subject to

$$\begin{split} \sum_{j \in J} (As_j + Bh_j), \\ \sum_{k \in \tilde{\mathcal{K}}} z_{jk} &= 1, \qquad j \in \mathcal{J}, \\ z_{jk} &\leq \lambda_{jk}, \qquad j \in \mathcal{J}, \ k \in \tilde{\mathcal{K}}, \\ y_{jqk} + y_{qjk} \leq z_{jk}, \qquad j, q \in \mathcal{J}, \ j \neq q, \ k \in \tilde{\mathcal{K}}, \\ y_{jqk} + y_{qjk} + 1 \geq z_{jk} + z_{qk}, \qquad j, q \in \mathcal{J}, \ j \neq q, \ k \in \tilde{\mathcal{K}}, \\ t_j + p_j - t_q &\leq M(1 - y_{jqk}), \qquad j, q \in \mathcal{J}, \ j \neq q, \ k \in \tilde{\mathcal{K}}, \\ t_j \geq r_j^{\mathfrak{m}}, \qquad j \in \mathcal{J}, \\ t_j \geq a_k z_{jk}, \qquad j \in \mathcal{J}, \\ t_q \geq s_j + v_{jq}^{\mathfrak{m}}, \qquad (j,q) \in \mathcal{Q}, \\ s_j - t_j = p_j + p_j^{\mathfrak{pm}}, \qquad j \in \mathcal{J}, \\ h_j \geq \max\{s_j - d_j; 0\}, \quad j \in \mathcal{J}. \end{split}$$

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The engineer's model

Minimize

subject to

$$\begin{split} \sum_{j \in J} (As_j + Bh_j), & \text{weights (A=B=1)} \\ \sum_{z j k} z_{j k} &= 1, \\ z_{j k} &\leq \lambda_{j k}, \\ y_{j q k} + y_{q j k} &\leq z_{j k}, \\ y_{j q k} + y_{q j k} &+ 1 \geq z_{j k} + z_{q k}, \\ t_j + p_j - t_q &\leq M(1 - y_{j q k}), \\ t_j &\geq r_j^{\texttt{m}}, \\ t_j &\geq a_k z_{j k}, \\ t_q &\geq s_j + v_{j q}^{\texttt{m}}, \\ s_j - t_j &= p_j + p_j^{\texttt{pm}}, \\ h_j &\geq \max\{s_j - d_j; 0\} \end{split}$$

Objective: Minimize the weighted sum of completion times and tardiness.

One job is scheduled only once

Each job can only be assigned to an allowed resource k

Only one job at a time can be processed on resource *k*

Job j not allowed to start before its release date or before resource k is available

Planned lead time between jobs on same part

Definition of completion time

Definition of tardiness

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Comparison between the precedence constraints for the set ${\cal Q}$

$$\sum_{k \in \tilde{\mathcal{K}}} \left(\sum_{\mu=0}^{u} x_{jk\mu} - \sum_{\nu=0}^{u+\tilde{v}_{jq}^{pm}} x_{qk\nu} \right) \ge 0, \qquad t_q \ge s_j + v_{jq}^{m},$$

$$\sum_{k \in \tilde{\mathcal{K}}} \left(\sum_{\mu=0}^{u} x_{jk\mu} - \sum_{\nu=0}^{u+\tilde{v}_{jq}^{pm}} x_{qk\nu} \right) \ge 0, \qquad t_q \ge s_j + v_{jq}^{m},$$
Constraints: $|\mathcal{Q}| |\mathcal{T}| \qquad |\mathcal{Q}|$
Test case No 3: 372 4

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Comparison between the constraints for only one job at a time

 $y_{jqk} + y_{qjk} \leq z_{jk},$ u $\sum_{j\in\mathcal{J}}\sum_{\nu=(u-\tilde{p}_j+1)_+}x_{jk\nu}\leq 1,$ $y_{jqk} + y_{qjk} + 1 \ge z_{jk} + z_{qk},$ $t_j + p_j - t_q \le M(1 - y_{jak})$

Symmetric constraints versus ordering constraints (the engineer's model)

- The symmetric constraints are common in text books since the ordering constraints are special for problems with multiple machines.
- Computational tests indicate that the model with ordering constraints has **shorter computation times** than a model using the symmetric big *M*-constraints.

What is the size of a realistic scenario? 45 jobsNumber of storage locations without fixture:30Number of parts arriving during the coming shift:15

Real production scenarios

- 6 real scenarios based on real production data extracted from the Volvo Aero ERP-system during the autumn of 2010
- The jobs were ordered according to increasing release dates
- From each scenario test instances were created with 5,10,15,..., 70 of the first jobs in the sorted list of jobs (i.e. the queue of jobs)

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Realistic release dates

The release date from the ERP system may be negative (i.e. in the past). Therefore a realistic estimate on the part arrival time at the multitask cell is calculated using the knowledge of the part's actual position at time t_0 .

r_i = max {*realistic estimate; ERP release date*}

If a part is present in the multitask cell, i.e., if it is checked-in: $r_i = 0$

About the time horizon

A major disadvantage of the time-indexed model is that the amount of variables and constraints are dependent on the choice of the time horizon and the length of the time interval.

- A heuristic has been developed in order to determine a good value on the time horizon.
- What are good values for the time horizon and the length of the time interval is dependent on the instance data.

Postprocessing with real "undiscrete" data

Mean differences between optimal objective values after postprocessing of data

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Mean computation times

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Evolution of computation times

Comparison between different models' CPU times (seconds)

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Current detail planning of the multitask cell

Manual planning based on

- Earliest Due Date (EDD) priority list
- Other priorities based on the current logistical situation
- The FIFO priority rule (First In First Out) is used in other parts of the factory
- SPT (shortest processing time) is a priority rule known to produce good schedules

The deburring and set-up stations are scheduled by the use of the feasibility model.

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EDD, FIFO and SPT versus mathematical optimization

21 scenarios with real production data Collected during April – August 2010

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Work load variation

The variation in number of jobs checked-in indicate how the work load has varied during the period.

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Mean differences between completion times and tardiness results

The schedules resulting from the use of the priority rules are compared to the optimal values found by the time-indexed model with l=1h in sequence with the feasibility model. 22% higher tardiness

Shortsighted scheduling

No knowledge about which jobs are on the way to the multitask cell (or further down in the priority list)

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Looking into the future...

The optimization model takes all jobs in the queue into account

Looking into the future...

Looking into the future...

An optimal schedule versus a schedule created using the EDD rule

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Coping with reality

As soon as the production schedule is optimized – something changes!

Expected events

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- > New parts in the queue
- > Variances in the planned lead time

Coping with reality

As soon as the production schedule is optimized – something changes!

Expected events

- > New parts in the queue
- > Variances in the planned lead time

Unexpected events

- > Machine breakdown
- > Operator sick
- > Part with non-conformance leaves queue
- > etc.

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Future research

- Compare results with more sophisticated scheduling algorithms
- > More realistic model: Include unmanned time windows in a model for all resources in the cell together with the scheduling of maintenance actitivities and fixture availability
- > Constraint programming

