Questions on the network design problem

to be discussed at the workshop on Wednesday 24 January

Be prepared for this occasion

Each student must contribute to the discussion

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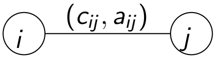
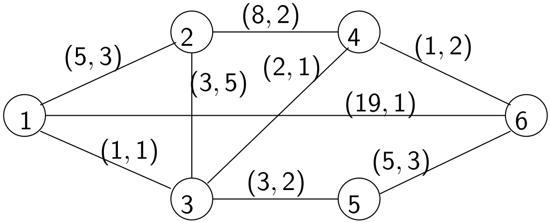
2018–01–22

Questions on the network design problem

1. Formulate mathematically the minimum spanning tree (MST) problem as a network flow problem. [*Hint*: consider node 1 as a sink and all other nodes as sources with strength 1.]

2. Consider the graph below.





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2. (a) Provide *several* the spanning trees of this graph explicitly. Why shouldn't one try to find all of them?
   (b) Calculate the sum of c_{ij} and a_{ij} for each of the trees from (a). Which of the trees are feasible with respect to the *budget constraint*
   $$\sum_{(i,j) \in \mathcal{T}} a_{ij} \leq 10 \quad ?$$
   [Here, $\mathcal{T}$ denotes a collection of links forming a spanning tree.] Which of the trees are optimal (minimal) with respect to the link costs c_{ij}?
   (c) Utilize the solution in (a) to formulate the MST problem with a budget constraint for a general graph.
   (d) Formulate the MST problem as a binary (integer) linear optimization problem. [*Hint*: Compare with integer linear optimization models for an undirected TSP.]
   (e) Is there a polynomial algorithm for the problem in (c)? [*Hint*: utilize that the binary knapsack problem is hard.]

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3. Formulate a polynomial *heuristic*, which provides a feasible solution to the MST problem with a budget constraint.
4. A *local search heuristic* finds the best solution in a *neighbourhood* of the current solution:
   i. If the new solution has a lower objective value than the current, choose the new solution to be the current; repeat
   ii. Otherwise, the current solution is locally optimal

   A *neighbourhood* of a tree is formally defined as (here $k \geq 1$)

   $$\mathcal{N}(\mathcal{T}) = \{\mathcal{S} \subseteq E \;:\; |\mathcal{S} \cap \mathcal{T}| \geq |V| - k, \mathcal{S} \text{ defines a tree in the graph } (V, E)\}.$$

   Provide a *local search* heuristic which improves a feasible solution to the MST problem with a budget constraint.
5. Provide a *Lagrangian relaxation* algorithm for the MST problem with a budget constraint.
   (a) Suggest a suitable relaxation.
   (b) How can/should the subproblems be solved?
   (c) Suggest a primal feasibility (local search) heuristic.
   (d) Provide a complete Lagrangian relaxation scheme.