# Trapetsformeln

## Trapetsformeln

Dela upp $[a, b]$ i $n$ delintervall med längd $h = (b - a)/n$ och definiera punkterna $x_i = a + ih$, $i = 0, \ldots, n$.
Trapetsformeln

$$T(h) = h\Big(\underbrace{\frac{f(x_0)}{2} + \frac{f(x_1)}{2}}_{\text{intervall 1}} + \underbrace{\frac{f(x_1)}{2} + \frac{f(x_2)}{2}}_{\text{intervall 2}} + \cdots + \underbrace{\frac{f(x_{n-1})}{2} + \frac{f(x_n)}{2}}_{\text{intervall } n}\Big)$$

$$= h\Big(\frac{f(x_0)}{2} + \frac{f(x_n)}{2} + \sum_{i=1}^{n-1} f(x_i)\Big)$$

approximerar integralen $\int_a^b f(x)dx$ med trunkeringsfelet

$$R_T = \frac{b-a}{12} h^2 f''(\xi), \text{ för något } \xi \in [a, b]$$

## Simpsons formel

Dela upp $[a, b]$ i $n$ delintervall med längd $h = (b - a)/n$ och definiera punkterna $x_i = a + ih$, $i = 0, \dots, n$. ($n$ jämnt tal)
Simpsons formel

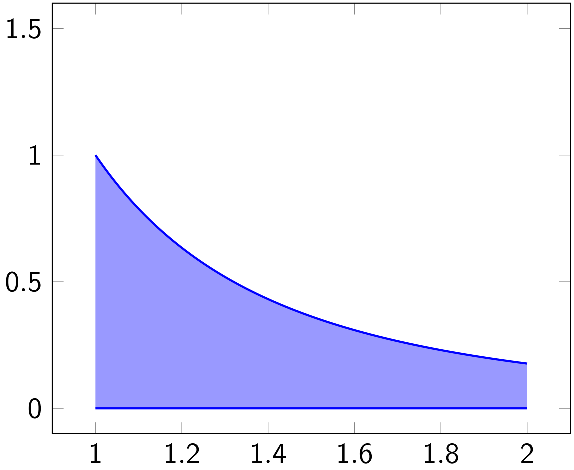
$$S(h) = \frac{h}{3}\Big(\underbrace{f(x_0) + 4f(x_1) + f(x_2)}_{\text{intervall 1\&2}} + \underbrace{f(x_2) + 4f(x_3) + f(x_4)}_{\text{intervall 2\&3}} + \cdots + 4f(x_{n-1}) + f(x_n)\Big)$$

$$= \frac{h}{3}\Big(f(x_0) + f(x_n) + 4\sum_{i=1}^{n/2} f(x_{2i-1}) + 2\sum_{i=1}^{n/2-1} f(x_{2i})\Big)$$

approximerar integralen $\int_a^b f(x)dx$ med trunkeringsfelet

$$R_T = \frac{b-a}{180} h^4 f^{(4)}(\xi), \text{ för något } \xi \in [a, b]$$

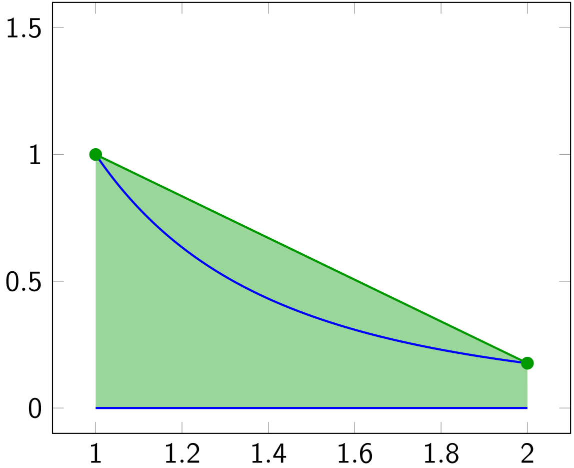
# Trapetsformeln




$$f(x) = x^{-5/2}$$

$$\int_1^2 f(x)dx = \frac{1}{6}\left(4 - \sqrt{2}\right) \approx 0.430964$$

# Trapetsformeln



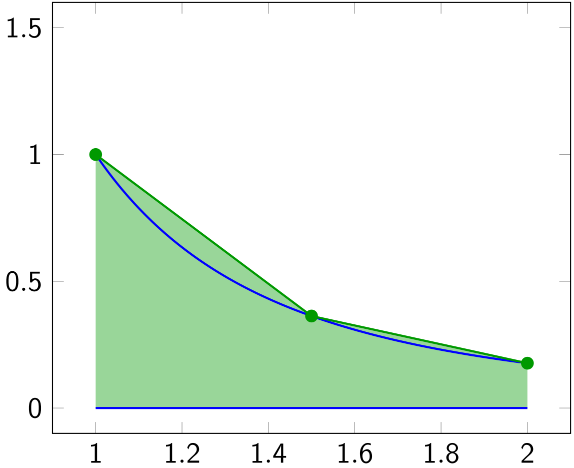

$$f(x) = x^{-5/2}$$

$$\int_1^2 f(x)dx = \frac{1}{6}\left(4 - \sqrt{2}\right) \approx 0.430964$$

Trapets $n = 1$

$$\int_1^2 f(x)dx = \frac{1}{2}\left(1 + \frac{1}{4\sqrt{2}}\right) \approx 0.588388$$

# Trapetsformeln



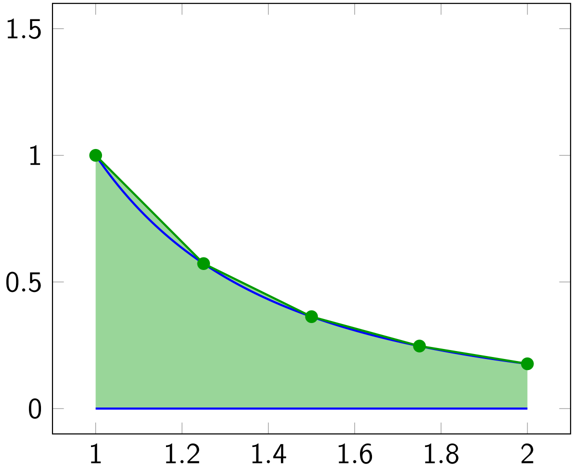

$$f(x) = x^{-5/2}$$

$$\int_1^2 f(x)dx = \frac{1}{6}\left(4 - \sqrt{2}\right) \approx 0.430964$$

Trapets $n = 2$

$$\int_1^2 f(x)dx \approx 0.475638$$

# Trapetsformeln



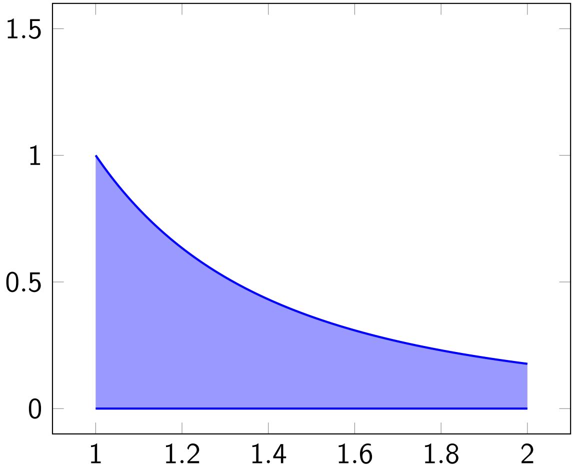

$$f(x) = x^{-5/2}$$

$$\int_1^2 f(x)dx = \frac{1}{6}\left(4 - \sqrt{2}\right) \approx 0.430964$$

Trapets $n = 4$

$$\int_1^2 f(x)dx \approx 0.442636$$

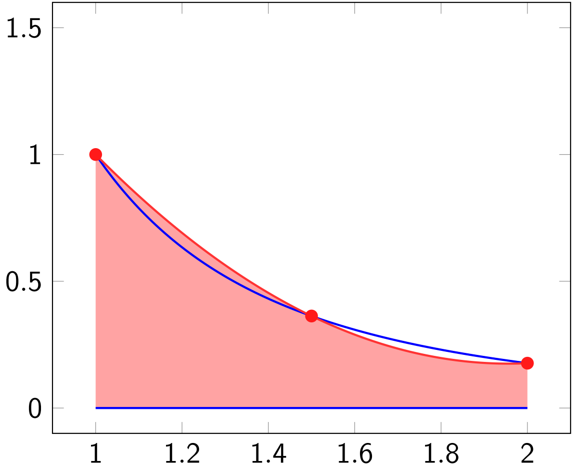
# Simpsons formel




$$f(x) = x^{-5/2}$$

$$\int_1^2 f(x)dx = \frac{1}{6}\left(4 - \sqrt{2}\right) \approx 0.430964$$

# Simpsons formel



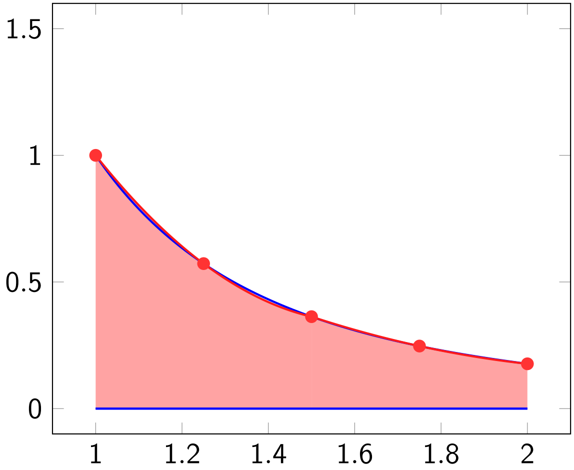

$$f(x) = x^{-5/2}$$

$$\int_1^2 f(x)dx = \frac{1}{6}\left(4 - \sqrt{2}\right) \approx 0.430964$$

Simpson $n = 2$

$$\int_1^2 f(x)dx \approx 0.438054$$

# Simpsons formel



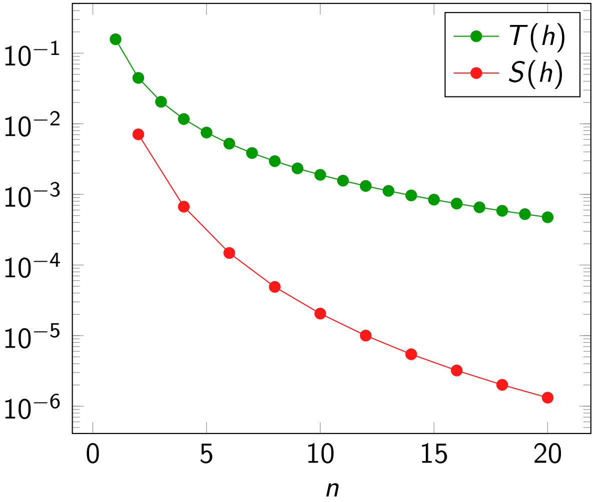

$$f(x) = x^{-5/2}$$

$$\int_1^2 f(x)dx = \frac{1}{6}\left(4 - \sqrt{2}\right) \approx 0.430964$$

Simpson $n = 4$

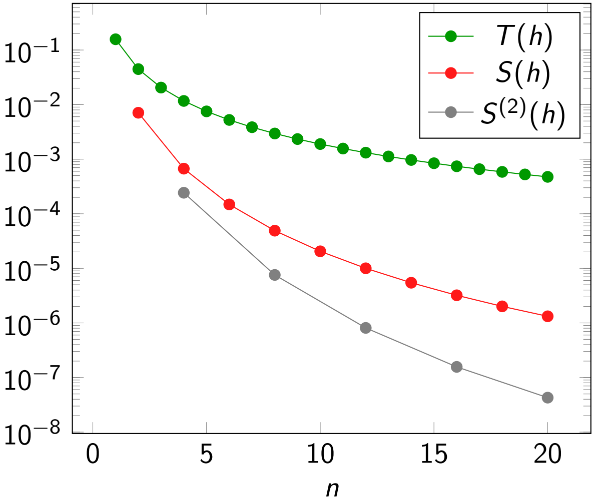
$$\int_1^2 f(x)dx \approx 0.431635$$

# Trunkeringsfel




För samma antal funktionsberäkningar ger Simpsons formel bättre resultat jämfört med trapetsformeln.

# Bonus: Richardsonextrapolation




Ännu mindre fel kan fås av uppskattningen

$$S^{(2)}(h) = S(h) + \frac{S(h) - S(2h)}{15}$$

Fungerar på grund av

$$S(h) = \int_a^b f(x)dx + a_1 h^4 + a_2 h^6 + \mathcal{O}(h^8)$$

$$S(2h) = \int_a^b f(x)dx + 2^4 a_1 h^4 + 2^6 a_2 h^6 + \mathcal{O}(h^8)$$

$$S^{(2)}(h) = \int_a^b f(x)dx - \frac{16}{15} a_2 h^6 + \mathcal{O}(h^8)$$

Tekniken kallas Richardsonextrapolation