

## LARGE AND SPARSE MATRIX PROBLEMS, 2009

### HOMEWORK ASSIGNMENT number 2

Well performed this homework assignment gives 1 credit point

To be handed in by February 11 at the latest

**Exercise HA2 a.** Derive the Lanczos algorithm (Alg. 6.10) for a symmetric matrix from the sequence of vectors in (6.29) by means of a QR factorization of  $K$ . The order in which the elements of  $Q$  and  $T$  are calculated shall be made clear. (0.5 point)

**Exercise HA2 b.** Solve Question Q6.13 in the text book. (0.5 point)

### COMPUTER EXERCISE number 2

To be handed in by February 11 at the latest

**Exercise CE2.** We will study the preconditioned conjugate gradient method from the rate of convergence point of view. For this purpose we use the MATLAB function *pcg*. We let the preconditionings be based on incomplete Cholesky factorization of symmetric matrices. There are two kinds of such factorizations, based on a dropping tolerance or based on restricted fill-in in beforehand selected subdiagonals. The MATLAB function *cholinc* takes the first approach and you can choose the degree of fill-in by a parameter. For the other approach you can use the routines called *ic* or *mic* available from the course web site, where *m* stands for modification. The idea behind modification is to keep the rowsums of the matrix unchanged and this modification is very easy to implement and gives, as you will find, a significant increase in the rate of convergence. MATLAB also offers modification as an option, but it is not correctly implemented, as you could find out.

Use the Poisson's model problems in two and three dimensions as test problems. For the 3D problem you may use the function *delsq3d* from the course web page. You have 2D and 3D versions of the *ic* and *mic* methods and the letter *d* in *icd* and *micd* stands for the number of filled-in subdiagonals in the incomplete Cholesky factor. Choose a constant right hand side or take it to be zero except in one or a few appropriately chosen points, think that we (in 2D) model a tent with supporting poles in some positions. Choose stepsize  $h$  and residual stopping criterion in order to be able to confirm the theory by the experiments. Compute condition numbers by *eigs* and study the sparsity by *nnz* and *spy*.

Draw conclusions and give some advice about choice of methods for the problems studied.

**About grades.** This CE is graded according to how well the discussion is performed regarding confirming the theory by the experiments and the way you support your advice for choice of methods.