Planning of the Mexican steel production, part II: The optimal plan, with a sensitivity analysis

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Introduction

The Mexican government were very happy with your report on how to optimize their steel production. Therefore, they want you to be responsible for the planning of the steel production during 2006. You have accepted their offer.

Your intention is, of course, to use your linear programming model to solve the problem (assume that the data given corresponds to 2006). However, in order to be able to give directions to the managers of the different mines and steel mills you must numerically solve the model! To your help you have the modelling language AMPL and the linear programming solver CPLEX.

During the coming year you will write reports with directions to all of the managers involved in the steel production, but already some of the managers are interested in what will happen. In particular, you have received the questions according to the following sections. Your task is to answer these questions. (You only have to answer *one* of the questions in "Questions, part III".)

Questions, part I

Answer the following questions about the optimal solution, assuming that export and import are not allowed, that is, let the import- and export-limits in the AMPL-model be zero.

1. Basic questions:
   (a) What is the total cost of the steel production in 2006?
   (b) How much iron ore should be sent from the ore mine in Laperla to the steel mill in Sicartsa?
   (c) How much steel will be produced by the Martin-oven at the steel mill in Ahmsa?
   (d) How much plates will be sent from the steel mill in Hylsap to Mexico City?

2. Questions regarding the capacities:

   (a) Which mines have reached their capacities? Explain how to identify such mines by using the values of the dual variables corresponding to the capacity constraints together with the Complementary Slackness Theorem of linear programming!

   (b) Which processes have reached their capacities?

   (c) Which steel mills have reached their capacities? (Observe that some of the mills have parallel processes and then the capacity is reached only if the capacities of both of the processes are reached.)

   (d) How much would we gain from a marginal improvement of the limiting capacities? Answer this question by making small increments (one at a time) of the limiting capacities and resolve the model to see how the total cost changes. Compare the result with the values of the dual variables for the limiting capacities in the original problem. (Remember that not every process is present at every mill, and we may only expand existing processes, that is, processes with a maximum capacity greater than 0.)

Questions, part II

The following questions deal with export and import:

1. Assume that export is allowed and unlimited. (But import is not allowed.) That is, you must set the limit of export to a high value in the AMPL-model.

   (a) By using the dual variable of the limiting constraint for the export in the AMPL-model, find the price in pesos per ton at which it starts to pay off to export products, and denote this price by P_{export}.

   (b) Let the price of export be the price found above plus 1 peso per ton, that is, $P_{\text{export}} + 1$. How much will be exported at this price?

2. Assume that import is allowed and unlimited. (But export is not allowed.)

   (a) In the same way as for the export, find the price in pesos per ton at which it starts to pay off to import products, and denote this price by P_{import}.

   (b) Let the price of import be the price found above minus 1 peso per ton, that is, $P_{\text{import}} - 1$. How much will be imported at this price?

Questions, part III

The following questions deal with sensitivity analysis. You shall compare the solution found with the original data (assumed that export and import are not allowed), with the solution you get after the specific change in the data. You only have to answer *one* of the four questions. Choose the one that you find the most interesting!

1. Assume that the price of energy is increased by a factor of 10. How does the solution change? In particular, answer the following questions:
   (a) How does the total production of steel change at the steel mill in Ahmsa?
   (b) How does the total production of steel change at the steel mill in Hylsa?
   (c) Explain why the above changes occur!
2. Assume that the proportional cost of transporting raw materials from all mines is increased by a factor of 3. How does the solution change? In particular, answer the following questions:
   (a) How does the total production of steel change at the steel mill in Ahmsa?
   (b) How does the total production of steel change at the steel mill in Fundidora?
   (c) Explain why the above changes occur!
3. Assume that the distance between Penacol and Ahmsa is decreased by 1 200 km (through the building of a tunnel). How does the solution change? In particular, answer the following questions:
   (a) How does the total production of steel change at the steel mill in Ahmsa?
   (b) How does the total production of steel change at the steel mill in Fundidora?
   (c) Explain why the above changes occur!
4. What is the gain from investing in new technology? Assume that we get a loan from the IMF to modernize the steel mill in Ahmsa by building a facility for reducing ore to sponge iron, and an electro oven, both limited to 0.5 Mton output. How does the solution change? In particular, answer the following questions:
   (a) How does the production of steel by the Martin-oven change at the steel mill in Ahmsa?
   (b) How does the transportation of coal to the steel mill in Ahmsa change?
   (c) Explain why the above changes occur!