EXERCISE 5: KARUSH-KUHN-TUCKER OPTIMALITY CONDITIONS

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EXERCISE 1 (KKT-conditions: Finding optimal solutions). Find the area of the largest isosceles triangle that is contained in the unit circle. □

EXERCISE 2 (KKT-conditions: Finding optimal solutions). Consider the problem to

$$\begin{aligned} \text{minimize} \quad & f(\boldsymbol{x}) = \sum_{j=1}^{n} c_j x_j \\ \text{subject to} \quad & \sum_{j=1}^{n} x_j^2 \leq 1, \\ & x_j \geq 0, \quad j = 1, \dots, n, \end{aligned}$$

where $\min_{j=1,\dots,n}\{c_j\} < 0$. Find an optimal solution to the problem! □

EXERCISE 3 (KKT-conditions: Finding optimal solutions). Consider the problem to

$$\begin{aligned} \text{minimize} \quad & f(\boldsymbol{x}) = \sum_{j=1}^{n} c_j x_j^2 \\ \text{subject to} \quad & \sum_{j=1}^{n} x_j = b, \end{aligned}$$

where b and c_j are all strictly positive constants. Find an optimal solution to the problem and show that it is unique! □

EXERCISE 4 (KKT-conditions: Investigating feasible solutions). Consider the problem to

$$\begin{aligned} \text{minimize} \quad & f(\boldsymbol{x}) = 4x_1^2 + 2x_2^2 - 6x_1x_2 + x_1 \\ \text{subject to} \quad & -2x_1 + 2x_2 \geq 1, \\ & 2x_1 - x_2 \leq 0, \\ & x_1 \leq 0, \\ & x_2 \geq 0. \end{aligned}$$

Is $\boldsymbol{x} = (0, 1/2)^{\mathrm{T}}$ a KKT point? Can you draw any conclusions from this regarding optimality of $\boldsymbol{x}$? □

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Exercise 5 (KKT-conditions: Investigating feasible solutions). Consider the problem to

$$
\begin{aligned}
\text{minimize} \quad & f(\boldsymbol{x}) = x_1^2 + 3x_2^2 - x_1 \\
\text{subject to} \quad & x_1^2 - x_2 \leq 1, \\
& x_1 + x_2 \geq 1.
\end{aligned}
$$

Is the point $\boldsymbol{x} = (1, 0)^{\mathrm{T}}$ a global minimum? □