EXERCISE 6: LAGRANGIAN DUALITY

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EXERCISE 1 (Formulating the Lagrangian dual problem). Consider the problem to

$$
\begin{aligned}
\text{minimize} \quad & f(\boldsymbol{x}) = x_1 + 2x_2^2 + 3x_3^3 \\
\text{subject to} \quad & x_1 + 2x_2 + x_3 \leq 3, && (1) \\
& 2x_1^2 + x_2 \geq 2, && (2) \\
& 2x_1 + x_3 = 2, && (3) \\
& x_1, \quad x_2, \quad x_3 \geq 0.
\end{aligned}
$$

(a) Formulate the Lagrangian dual problem that originates from a relaxation of the constraints (1)–(3).
(b) State the primal-dual optimality conditions!

□

EXERCISE 2 (Formulating the Lagrangian dual problem). Consider the linear program

$$
\begin{aligned}
\text{minimize} \quad & z = \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x} \\
\text{subject to} \quad & \boldsymbol{A}\boldsymbol{x} \geq \boldsymbol{b}, && (1) \\
& \boldsymbol{x} \geq \boldsymbol{0}^n,
\end{aligned}
$$

where $\boldsymbol{A} \in \mathbb{R}^{m \times n}$ and $\boldsymbol{b} \in \mathbb{R}^m$. Formulate the Lagrangian dual problem that originates from a relaxation of the constraints (1). □

EXERCISE 3 (Primal-dual optimality conditions: Finding optimal solutions). Consider the problem to

$$
\begin{aligned}
\text{minimize} \quad & f(\boldsymbol{x}) = x_1^2 + 2x_2^2 \\
\text{subject to} \quad & x_1 + x_2 \geq 2, \\
& x_1^2 + x_2^2 \leq 5.
\end{aligned}
$$

Find an optimal solution! □

EXERCISE 4 (Primal-dual optimality conditions: Finding optimal solutions). Consider the problem to

$$
\begin{aligned}
\text{minimize} \quad & f(\boldsymbol{x}) = \frac{1}{2}\|\boldsymbol{y} - \boldsymbol{x}\|^2 \\
\text{subject to} \quad & \boldsymbol{A}\boldsymbol{x} = \boldsymbol{0}^m,
\end{aligned}
$$

where $\boldsymbol{y} \in \mathbb{R}^n$ and $\boldsymbol{A} \in \mathbb{R}^{m \times n}$ such that rank $\boldsymbol{A} = m$. Find an optimal solution! □

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EXERCISE 5 (Primal-dual optimality conditions: Investigating feasible solutions). Consider the problem to

$$\begin{aligned} \text{minimize} \quad & f(\boldsymbol{x}) = -x_1 + x_2 \\ \text{subject to} \quad & x_1^2 + x_2^2 \leq 25, \\ & x_1 - x_2 \leq 1. \end{aligned}$$

Is the point $\boldsymbol{x} = (4, 3)^{\mathrm{T}}$ a global minimum? □