Manufacturing thickeners, part II

Finding and analyzing the optimal plan

5th January 2007

1 Introduction

The managers of Aditiva were very happy with your report on how to model their production system. Hence, they have decided to give you the job of finding the optimal plan for the year 2007. Without hesitation you decided to accept their offer.

Your intention is to use your linear programming model to solve the problem (assuming that the data given corresponds to 2007). However, to be able to give concrete proposals to the managers of Aditiva you must now solve the problem numerically. To your help you have the modelling language AMPL[1] and the linear programming solver CPLEX[2].

During the coming year you will write reports with directions to all the managers involved in the thickeners production, but already some of them are interested in what will happen. In particular, you have received the questions according to the following sections. Your task is to answer these questions. All questions in part A have to be answered, while in part B it is enough to answer two sets of questions.

Note that small numbers returned by AMPL are likely to be round-off errors.

Motivate your answers carefully. The explanations are important.

2 Questions, part A

Answer the following questions about the optimal solution. For the first set of questions, leave the model unchanged. To answer some questions in the second set of questions, small adjustments to the model have to be done. A suggestion is to use a `.run`-file in AMPL and to leave the `.mod`-file and the `.dat`-file unchanged[3]. All questions are per year.

1. Basic questions:
   (a) What is the profit when using the optimal plan?
   (b) How much NaOH is bought from the supplier in Vårgårda to be used in the reactor in Billdal?
   (c) How much CellOH is bought from the supplier in Lerum to be used in the reactor in Gamlestaden?
   (d) How much MC is dried using the fluidized bed-technique?
   (e) How much of raw product EC is being sold?
2. Questions regarding the capacities:
   (a) Which reactors have reached their capacities of handling Cellulose? Explain how to identify such reactors by using the values of the dual variables corresponding to the capacity constraints together with the Complementary Slackness Theorem of linear programming.

[1] www.ampl.com
[2] www.ilog.com/products/cplex/
[3] See the document `amplaid.pdf`

(b) Which demands are fulfilled? Explain briefly why not all of them have to be fulfilled.

(c) How much would Aditiva gain from marginal improvements of the reactor capacities? That is, if the revenue is denoted z and the different capacities c_i, find $\frac{\partial z}{\partial c_i}$, $i = 1 \ldots 6$ (2 at each reactor). Answer this question by making small increments of the capacities and then resolve the model. Compare the results with the values of the dual variables corresponding to the capacity constraints in the original model. (Note that the capacities are defined per day). What do you see?

3 Questions, part B

The following questions deal with sensitivity analysis. You shall compare the solution found with the original data with the solution you get with a specific change in the data. In the report you only have to answer two sets of questions. Pick one set of your own choice and let the other set be given by the formula $\bmod(x, 5) + 1$, where x is the sum of the last digits in the social secutiry numbers (personnummer) of the people in your group. Of course we encourage you to solve all problems, but include only two in the report.

1. Assume a dramatic increase in oil prices, leading to that all transport costs are multiplied by a factor of 10. How does the solution change? In particular, answer the following questions:

   (a) How does the total revenue change?

   (b) Whats happens to the total amount of raw material bought?

   (c) Explain why the above changes occur.

2. Assume that a new media report shows that thickeners dried with the fluidized bed induce some health risks. The total demands of the end products remain the same but the proportion of the qualities of both MC and EC are changed to 5% and 95% respectively. How does the solution change? In particular, answer the following questions:

   (a) How does the total revenue change?

   (b) What happens to the demand fulfillment on vacuum dried MC?

   (c) Explain why the above changes occur.

3. Assume that the Älvsborg bridge is blown to pieces by some previously unknown terrorist group and that this leads to increasing transport costs from the supplier in Torslanda to the reactors in Mölndal and Billdal by a factor of 2.

   (a) How does the total revenue change?

   (b) Which decision variables have different optimal values compared to the original solution?

   (c) Explain why the above changes occur. Motivate with geometrical arguments thinking of the feasible polehedron and the direction of the cost vector. What if the factor was 3 instead of 2?

4. What is the gain from investing in new technology? Assume that Aditiva gets a loan to modernize the reactor in Gamlestaden which after the modernization will double its capacities at the same time as the cost of producing, washing and purifying will decrease by 10%.
   (a) What is the total amount of raw material bought to be used in the reactor in Billdal?
   (b) What happens to the slacks in the constraints regarding ractor capacities?
   (c) Explain why the above changes occur.
   (d) How much is the investment worth for Aditiva, i.e. how much would they pay for it (each year)?
5. In the optimal solution to the original problem, nothing is bought from the supplier in Vårgårda. Assume now that Aditiva's good reputation leads to that the supplier is willing to lower his prices for Aditiva to get them as a customer, hoping that this will lead to good publicity effects that will attract other customers.
   (a) In particular, how much must the supplier lower his price on CellOH so that Aditiva will start buying from him?
   (b) If the net income is denoted f and the price on CellOH is denoted b, what kind of function is $f(b)$? Explain with a graph.