Makeup Exam for discrete mathematics D3
TMA965
August 19, 2003
Location: V
Time: 8:45-12:45
Jour: Jeff Steif 772 3513
Ingen hjalpmedel : No books or notes
For grades of 3,4,or 5, 12,18 or 24 needed.

1. (5 points).
Consider the ways to give 21 toys to 5 children.
(a). How many ways can this be done if children are distinguishable, toys are indistinguishable and there is a requirement that each child gets at least two toys?
(b). How many ways can this be done if both toys and children are distinguishable and there is a requirement that there are at least 3 children who each get at least one toy?

2. (5 points).
(a). Give an example of a graph which does not contain an Eulerian cycle. (Recall a cycle always ends at the same point that it starts and a Eulerian cycle is a cycle which goes through each edge exactly once.)
(b). What does Euler's theorem say about a sufficient and necessary condition for a graph to have an Eulerian cycle?
(c). Give an example of a graph which has chromatic number 3 and does not contain a triangle. Can you find such an example which is also a bipartite graph?

3. (5 points).
Consider the set of integers modulo 9000 with modular arithmetic. This is

the set $\{0, 1, \ldots, 8999\}$ together with both modular addition and modular multiplication.
(a). How many of these elements have additive inverses?
(b). Describe which elements have multiplicative inverses. How many are there?
(c) How does one compute the multiplicative inverse of an element (assuming it has a multiplicative inverse)?

4. (5 points).
(a). What is the cycle representation (i.e., the way of writing the permutation in terms of disjoint cycles) of the following permutation. $\pi(1) = 7, \pi(2) = 5, \pi(3) = 8, \pi(4) = 2, \pi(5) = 4, \pi(6) = 1, \pi(7) = 6$ and $\pi(8) = 3$.
(b). What is the order of the permutation π? In other words, how many times do you need to compose π with itself in order to get the identity permutation?
(c). How many fixed points (1-cycles) does π^2 have? (π^2 is the permutation obtained by composing π with itself.)
(d). Can π be expressed as a product of 3 transpositions? Can π be expressed as a product of 8 transpositions? (Recall that a transposition is a cycle of length 2).

5. (5 points).
Consider the group G of rigid motions of a square to itself. (The rigid motions include flipping over the square.)
(a). How many elements are there in this group?
(b). Consider the subset H of G consisting of elements which send the left two points to themselves. Is H a subgroup of G? How many elements does H have?
For the rest of the problem, think of G and H as groups of permutations of the 4 corners (or vertices) of the square.
(c). How many elements of G are there in the stabilizer of a fixed corner (or vertex) v? (Recall that the stabilizer of an element is the set of all permutations in the permutation group which fix the element.) How many orbits are there for the corner points when we consider the permutations in G?
(d). How many elements of H are there in the stabilizer of a fixed corner (or vertex) v? How many orbits are there for the 4 corners when we consider the permutations in H?

6. (5 points).
What is the last digit in the number 7^{111}? (Hint: use modular 10 arithmetic.)