Lösningsförslag till dugga 2, 20131121

Lösningsförslag till dugga 2 a

1. (a) ja

   (b) nej ($\displaystyle\int_0^\infty \frac{x\,dx}{x^2+1} = \lim_{b\to\infty} \frac{1}{2}\big(\ln(b^2+1) - \ln(0^2+1)\big) = \infty$.)

   (c) ja

   (d) nej

2. (a) $\displaystyle\int x\,\cos x\,dx = x\sin x + \cos x + C\,.$

   (b) $\displaystyle\int_0^1 \frac{x}{\sqrt{9-x^2}}\,dx = \Big[-\sqrt{9-x^2}\Big]_0^1 = 3 - 2\sqrt{2}\,.$

3.
$$\begin{aligned}\int_0^3 \frac{x+2}{\sqrt{x+1}}\,dx &= \int_0^3 \left(\frac{x+1}{\sqrt{x+1}} + \frac{1}{\sqrt{x+1}}\right) dx = \\ &= \int_0^3 \left(\sqrt{x+1} + \frac{1}{\sqrt{x+1}}\right) dx = \\ &= \left[\frac{2}{3}\,(x+1)^{3/2} + 2\,(x+1)^{1/2}\right]_0^3 = \frac{20}{3}\end{aligned}$$

Lösningsförslag till dugga 2 b

1. (a) nej

   (b) ja (Jämförelsekriterium: $0 \le \dfrac{x}{x^3+1} \le \dfrac{x}{x^3+0} = \dfrac{1}{-}x^2$ och $\displaystyle\int_1^\infty \frac{1}{x^2}\,dx$ konvergent.)

   (c) nej

   (d) nej

2. (a) $\displaystyle\int x\,\sin x\,dx = \sin x - x\cos x + C,$

   (b) $\displaystyle\int_0^3 \frac{x}{\sqrt{9+x^2}}\,dx = \Big[\sqrt{9+x^2}\Big]_0^1 = 3\sqrt{2} - 3\,.$

3.
$$\begin{aligned}\int_{-1}^2 \frac{x+1}{\sqrt{x+2}}\,dx &= \int_{-1}^2 \left(\frac{x+2}{\sqrt{x+2}} - \frac{1}{\sqrt{x+2}}\right) dx = \\ &= \int_{-1}^2 \left(\sqrt{x+2} - \frac{1}{\sqrt{x+2}}\right) dx = \\ &= \left[\frac{2}{3}\,(x+2)^{3/2} - 2\,(x+2)^{1/2}\right]_{-1}^2 = \frac{8}{3}\end{aligned}$$