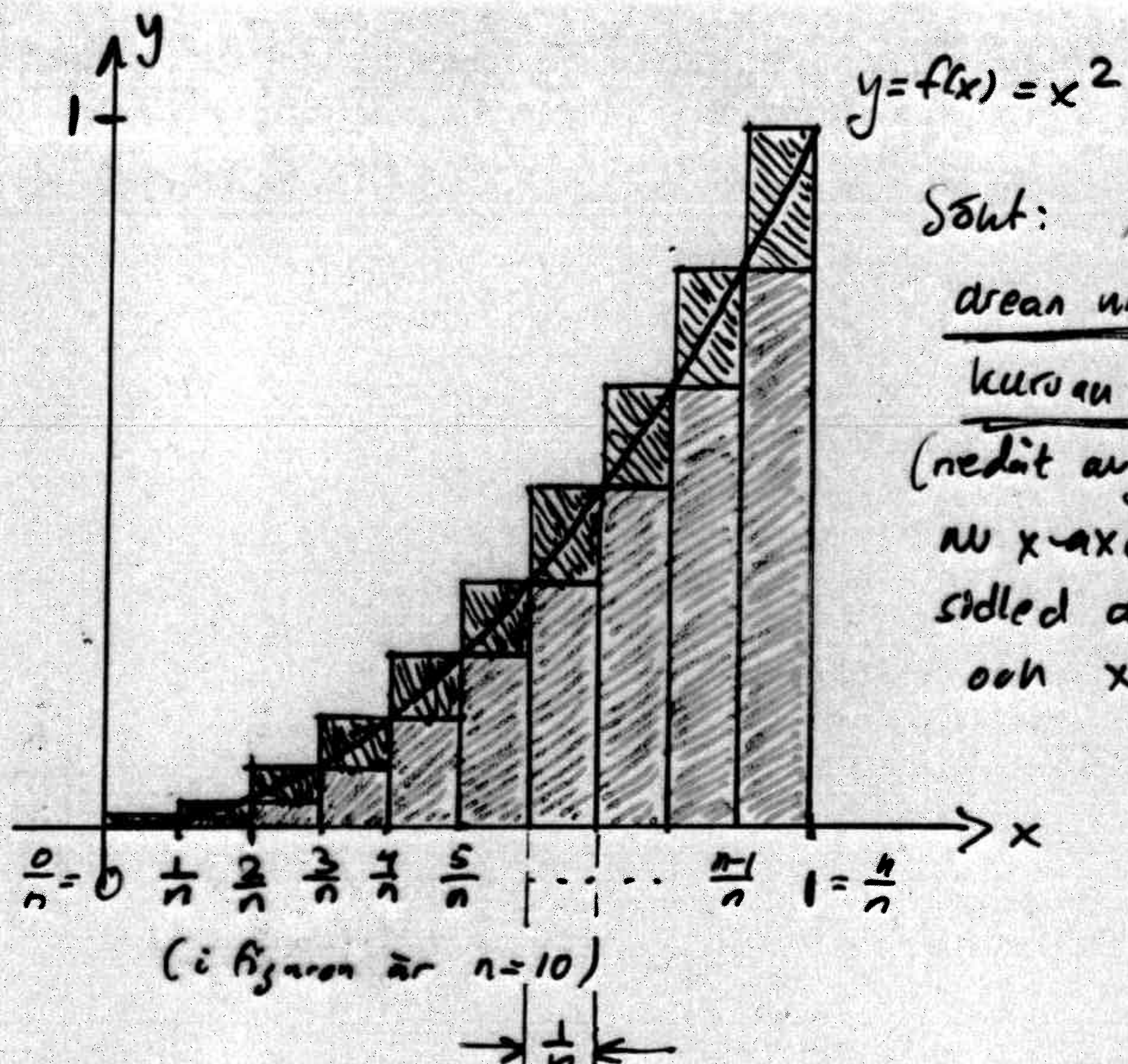

Sökt: $A =$ arean under kurvan $y=x^2$ (nedåt avgränsad av x-axeln, i sidled av $x=0$ och $x=1$)

Undersumma (gröna rektangelareor summeras):

$$\frac{1}{n} f(0)+\frac{1}{n} f\left(\frac{1}{n}\right)+\frac{1}{n} f\left(\frac{2}{n}\right)+\dots+\frac{1}{n} f\left(\frac{n-1}{n}\right)=$$

$$= \frac{1}{n}\cdot 0^2+\frac{1}{n}\left(\frac{1}{n}\right)^2+\frac{1}{n}\left(\frac{2}{n}\right)^2+\dots+\frac{1}{n}\left(\frac{n-1}{n}\right)^2=$$

$$=\frac{1}{n^3}\left(0^2+1^2+2^2+\dots+(n-1)^2\right)=\frac{1}{n^3}\sum_{k=0}^{n-1}k^2=\frac{1}{n^3}\cdot\frac{(n-1)\cdot n(2n-1)}{6}$$

$$=\frac{1}{6}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

Översumma (+ röda rektanglar):

$$\frac{1}{n} f\left(\frac{1}{n}\right)+\frac{1}{n} f\left(\frac{2}{n}\right)+\dots+\frac{1}{n} f\left(\frac{n}{n}\right)=\frac{1}{n^3}\sum_{k=1}^{n}k^2=\frac{1}{n^3}\frac{n(n+1)(2n+1)}{6}$$

$$=\frac{1}{6}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)$$

$$\underbrace{\frac{1}{6}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)}_{\to 1/3} < A < \underbrace{\frac{1}{6}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}_{\to 1/3 \text{ då } n\to\infty} \Rightarrow \boxed{A=1/3}$$