# Extra repetition 2014-10-09



## Definition: Gränsvärde

Funktionen $f$ har gränsvärdet $\bar{f}$ i punkten $\bar{x}$ om

(i) $\forall \varepsilon > 0 \; \exists \delta > 0 : 0 < |x - \bar{x}| < \delta \Rightarrow |f(x) - \bar{f}| < \varepsilon$

(ii) $\forall \varepsilon > 0 \; \exists \delta > 0 : \underset{(x \neq \bar{x})}{x \in B_\delta(\bar{x})} \Rightarrow f(x) \in B_\varepsilon(\bar{f})$

(iii) $x \to \bar{x} \Rightarrow f(x) \to \bar{f}$ (ej definition)

## Definition: Cauchy-följd $\{x_i\}$

$\forall \varepsilon > 0 \; \exists N > 0 : i, j \geq N \Rightarrow |x_i - x_j| < \varepsilon$

## Definition: Konvergent talföljd $\{x_i\}$

$\forall \varepsilon > 0 \; \exists N > 0 : i \geq N \Rightarrow |x_i - \bar{x}| < \varepsilon$

## Sats: Konvergens $\Leftrightarrow$ Cauchy

$\{x_i\}$ konvergent $\Leftrightarrow$ $\{x_i\}$ Cauchy

$|x_i - x_j| \leq |x_i - \bar{x}| + |\bar{x} - x_j|$

definition av $\mathbb{R}$

Övning 4: $\{x_i\}$, $\{y_i\}$ Cauchy, $|y_i| \geq \alpha > 0$

$\Rightarrow \{x_i / y_i\}$ Cauchy

Låt $z_i = x_i / y_i$.

$$|z_i - z_j| = |x_i / y_i - x_j / y_j| = \frac{|x_i \cdot y_j - x_j \cdot y_i|}{|y_i \cdot y_j|}$$

$$\leq \frac{1}{\alpha^2} \cdot |x_i y_j - x_j y_i|$$

$$= 1/\alpha^2 \cdot |x_i y_j - x_j y_j + x_j y_j - x_j y_i|$$

$$\leq 1/\alpha^2 \, |x_i - x_j| \cdot |y_j| + 1/\alpha^2 |x_j| \cdot |y_i - y_j|$$

Notera nu att $\{x_i\}$ Cauchy $\Rightarrow x_i \to \bar{x}$

$\Rightarrow |x_i| = |x_i - \bar{x} + \bar{x}| \leq |x_i - \bar{x}| + |\bar{x}| \leq 1 + |\bar{x}|$
för $i \geq N_x(1)$

På samma sätt: $|y_i| \leq 1 + |\bar{y}|$ för $i \geq N_y(1)$

$$\Rightarrow |z_i - z_j| \leq \frac{1}{\alpha^2} \cdot |x_i - x_j| \cdot (1 + |\bar{y}|) + \frac{1}{\alpha^2} (1 + |\bar{x}|) \cdot |y_i - y_j|$$

$$\leq \frac{1 + |\bar{y}|}{\alpha^2} \cdot \frac{\varepsilon/2 \cdot \alpha^2}{1 + |\bar{y}|} + \frac{1 + |\bar{x}|}{\alpha^2} \cdot \frac{\varepsilon/2 \cdot \alpha^2}{1 + |\bar{x}|}$$

$$= \varepsilon/2 + \varepsilon/2 = \varepsilon$$

om $i, j \geq \max\left(N_x(1), N_y(1), N_x\left(\frac{\varepsilon/2 \cdot \alpha^2}{1 + |\bar{y}|}\right), N_y\left(\frac{\varepsilon/2 \cdot \alpha^2}{1 + |\bar{x}|}\right)\right)$

Adams 4.9.16

$\sqrt{47} \approx ?$

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}$

$\Rightarrow \quad f(x) \approx f(\bar{x}) + f'(\bar{x}) \cdot (x - \bar{x})$

$= \sqrt{\bar{x}} + \frac{1}{2\sqrt{\bar{x}}} \cdot (x - \bar{x})$

Tag $\bar{x} = 49$

$\Rightarrow \quad \sqrt{47} = f(47) \approx \sqrt{49} + \frac{1}{2\sqrt{49}} \cdot (47 - 49)$

$= 7 + \frac{1}{14} \cdot (-2) = 7 - 1/7 = \underline{\underline{6\,6/7}}$

(Korrekt med 2 decimaler.)