ÖB. GAMLA DUGGOR

Testdugga 1 2013-2014

3. Visa att $\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

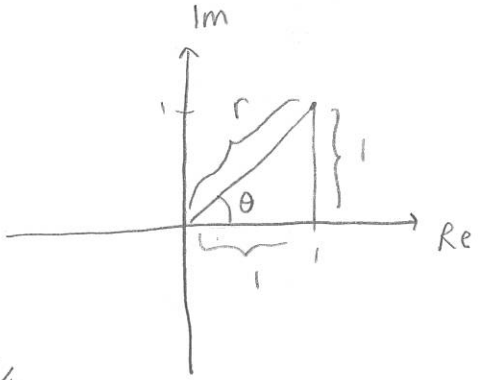
Använd $\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\tan x = \dfrac{\sin x}{\cos x}$

dividera med $\cos x, \cos y$

(om mult 2 komplexa tal adderas vinklarna och multipliceras längderna

4. Lös ekvationen $z^3 = 1+i$ och rita ut rötterna i det komplexa talplanet.



Skriv z på polär form, $z = re^{i\theta}$

skriv $1+i$ på polär form

$\arg(1+i) = \tan^{-1}(1) = \dfrac{\pi}{4}$

$|1+i| = \sqrt{1^2+1^2} = \sqrt{2} \Rightarrow 1+i = \sqrt{2}e^{i\pi/4}$

$\Rightarrow z^3 = r^3 e^{i3\theta} = \sqrt{2}e^{i\pi/4} = 2^{1/2}e^{i\pi/4}$

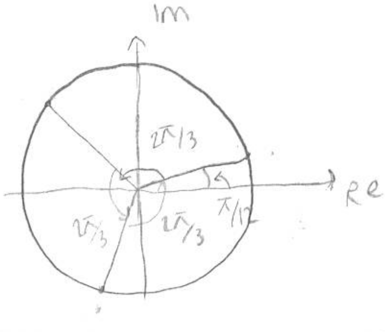
$\Rightarrow r^3 = 2^{1/2} \Rightarrow r = 2^{1/6}$

$e^{i3\theta} = e^{i\pi/4} \Rightarrow 3\theta = \dfrac{\pi}{4} + 2k\pi, \quad k = 0,1,2.$

$\theta = \dfrac{\pi}{12} + \dfrac{2}{3}k\pi, \quad k = 0,1,2$

$\theta_1 = \dfrac{\pi}{12}, \quad \theta_2 = \dfrac{9\pi}{12} = \dfrac{3\pi}{4}, \quad \theta_3 = \dfrac{17\pi}{12}$

Rita ut i talplanet



Alla ligger på cirkel runt origo med radien $2^{1/6}$ eftersom alla tre har samma längd.

$\dfrac{\pi}{12} = \dfrac{\pi}{2}/6$

$\dfrac{17\pi}{12} = \dfrac{16\pi}{12} + \dfrac{\pi}{12} = \dfrac{4\pi}{3} + \dfrac{\pi}{12}$

Testdugga 2

1. $$\frac{2x^2}{x+2} < x-2 \iff \frac{2x^2}{x+2} - (x-2) < 0 \iff \frac{2x^2}{x+2} - \frac{(x-2)(x+2)}{x+2} < 0$$

$$\iff \frac{2x^2 - (x^2-4)}{x+2} < 0 \iff \frac{x^2+4}{x+2} < 0$$

$x^2+4 > 0 \quad \forall x$

$x+2 < 0 \iff \underline{\underline{x < -2}}$

x		-2	
x^2+4	+	+	+
$x+2$	−	odef.	+

2. $$2x^3-4x^2-16x = x(2x^2-4x-16) = 2x(x^2-2x-8)$$

$$x^2-2x-8 = (x-1)^2-9 = 0 \iff x = 1 \pm 3 \quad x_1 = 4,\ x_2 = -2$$

(Faktorsatsen ger) $2x^3-4x^2-16x = 2x(x-4)(x+2)$

Kunde också se att $2x(x^2-2x-8) = 2x((x-1)^2-8-1) =$

$= 2x((x-1)^2-3^2) = \{\text{konjugatregeln}\} = 2x(x-1-3)(x-1+3)$

$= 2x(x-4)(x+2)$