Dugga 13/9 – 2014

F6.5

#tmv225

1.

$$\frac{a^6-8}{a^4+4a^2+4} = \{\text{låt } b = a^2\}$$

$$= \frac{b^3-2^3}{b^2+4b+4} = \frac{(b-2)(b^2+\textcircled{2}b+4)}{b^2+\textcircled{4}b+4}$$

∴ Svar: <u>D</u>

2.

$$\left(\sqrt{6-\underbrace{\sqrt{20}}_{<6}}\right)^2 = 6-\sqrt{20} = 6-\sqrt{4\cdot 5} = 6-2\sqrt{5}$$

a) $(\underbrace{\sqrt{5}-1}_{>0})^2 = 5+1-2\sqrt{5} = 6-2\sqrt{5}$, Ja

b) $1-\sqrt{5} < 0$, Nej

c) Nej

∴ Svar: <u>A</u>

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3. $\sqrt{x+3} = -x-3$, $x \geq -3$

$\Leftrightarrow \sqrt{x+3} = -(x+3)$, $x \geq -3$

En lösning: <u>$x=-3$</u>.

För $x > -3$:

$$1 = -\frac{x+3}{\sqrt{x+3}} = \underbrace{-\sqrt{x+3}}_{<0}$$ saknar lösning

∴ Svar: <u>B</u>

4.

$$4^x - 10 \cdot 2^x + 16 = 0$$

Notera: $4^x = (2^2)^x = 2^{2x} = (2^x)^2$

Låt $z = 2^x$

$$z^2 - 10 \cdot z + 16 = 0$$

$$z = 5 \pm \sqrt{25-16} = 5 \pm \sqrt{9} = 5 \pm 3$$

$$= \begin{cases} 8 \\ 2 \end{cases}$$

$2^x = 8 \iff x = \log_2 8 = 3$

$2^x = 2 \iff x = 1$

$\therefore$ Svar: C

5.

$$\log_3(\log_5 x) \leq 0$$

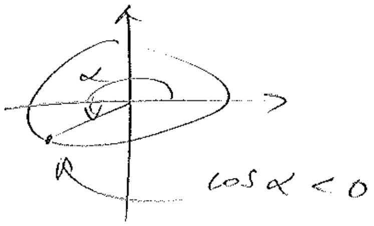
$\iff \log_5 x \leq 1$

$\iff 5^{\log_5 x} \leq 5^1 = 5$

$$x \leq 5$$

$\therefore$ Svar: C

6. $\sin\alpha = -\frac{1}{3}$, $\pi < \alpha < \frac{3\pi}{2}$



$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$= 2\cdot\left(-\frac{1}{3}\right)\left(-\sqrt{1-\frac{1}{9}}\right)$

$= 2\cdot\frac{1}{3}\cdot\sqrt{8/9} = \frac{2}{3}\cdot\frac{\sqrt{2\cdot 4}}{3}$

$= \frac{4\sqrt{2}}{9}$

$\therefore$ Svar: B

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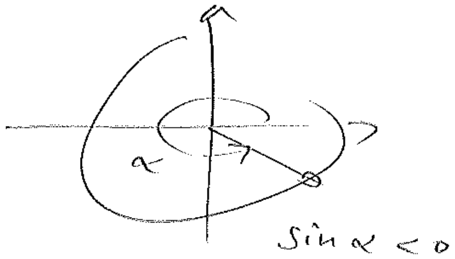
7. $\cos\alpha = t$, $\frac{3\pi}{2} < \alpha < 2\pi$



$\sin\alpha = -\sqrt{1-t^2}$

$\Rightarrow \tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{-\sqrt{1-t^2}}{t}$

$= \frac{\sqrt{1-t^2}}{-t}$

$\therefore$ Svar: C

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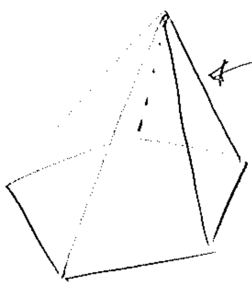
8. Pyramid: Polygonbas + spets



en kant från varje hörn

n-hörning : n st kanter

Totalt $n + n = 2n$ kanter (jämnt antal)

$\therefore$ Svar: 29

9. $$x^2 + 24xy + 68y^2 = 0$$

Notera: $x = y = 0$ är lösning

Ansätt $y = kx$:

$$x^2 + 24 \cdot x \cdot kx + 68(kx)^2 = 0$$

$$x^2 + 24kx^2 + 68k^2x^2 = 0$$

$x \neq 0$ ger

$$1 + 24k + 68k^2 = 0$$

$$k^2 + \frac{\overset{\cancel{12}\ 6}{\cancel{24}}}{\underset{\underset{17}{\cancel{34}}}{\cancel{68}}} k + \frac{1}{68} = 0$$

$$k = -\frac{6}{24} \pm \sqrt{\left(\frac{6}{24}\right)^2 - \frac{1}{68}} = k_1, k_2$$

$\therefore$ Två linjer $y = k_1x$, $y = k_2x$ genom origo.

Degenererad hyperbel

$\therefore$ Svar: $\underline{\underline{?}}$

10.

$$z + \bar{z} > 0 \Rightarrow \text{Re } z > 0$$

$$iz + \overline{iz} < 0 \Rightarrow \text{Im } z > 0$$

} Första kvadranten

$$iz + \overline{iz} = (a+bi)\cdot i + \overline{(a+bi)i}$$

$$= ai - b + \overline{ai - b}$$

$$= \cancel{a}i - b - \cancel{a}i - b$$

$$= -2b < 0 \iff 2b > 0 \iff b > 0$$

∴ Svar: A

11.

$z = 2e^{i\pi/3}$, $|z| = 2$, $z \in 1$:a kvadranten

a) $|e^{i2\pi/3}| = 1 \neq 2$

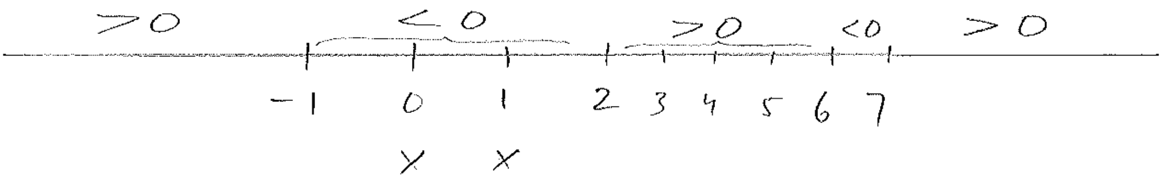
b) $-2e^{-i\pi/3} \in 2$:a kvadranten

c) $\overline{2e^{-i\pi/3}} = 2\cdot e^{i\pi/3} = z$

∴ Svar: C

12.

$$(x+1)(x-2)(x-6)(x-7) < 0$$



2 st.

∴ Svar: A

3.

$$x^2 + 2y^2 = 2$$

$x = a\cos t$

$y = b\sin t$

Alternativ lösning:

$\Rightarrow \frac{x^2}{2} + y^2 = 1$

$\left(\frac{x}{\sqrt{2}}\right)^2 + \left(\frac{y}{1}\right)^2 = 1$

$\rightarrow a = \sqrt{2}, \ b = 1$

om vi känner till parametrisering av ellips: $\begin{cases} x = a\cos t \\ y = b\sin t \end{cases}$

$$a^2\cos^2 t + 2b^2\sin^2 t = 2$$

Skall gälla för alla t.

Tag $a = 2b^2$

$\Rightarrow \cancel{2}b^2\cos^2 t + \cancel{2}b^2\sin^2 t = \cancel{2}$

$b^2 \cdot \underbrace{(\cos^2 t + \sin^2 t)}_{=1} = 1$

$\Rightarrow b^2 = 1 \Rightarrow b = (\pm)\,1 \rightarrow a^2 = 2 \Rightarrow a = (\pm)\sqrt{2}$

$\rightarrow$ Svar: D

14. $\frac{3-x}{x+2} > 0, \quad x \neq -2$

$\Leftrightarrow$ $(3-x)$ och $(x+2)$ har samma tecken och $x \neq -2$

$\Leftrightarrow$ $(x-3)$ och $(x+2)$ har olika tecken och $x \neq -2$

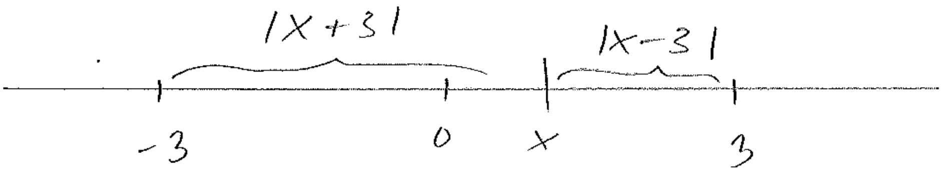
$\Rightarrow$ ekvivalent med a), b), c) $\not$ ty $= 0$ om $x = 2$

$\therefore$ Svar: D

15.

$$|x+3| + |x-3| = 6$$

$\uparrow$ avstånd till -3 $\qquad$ $\uparrow$ avstånd till 3



$\Rightarrow$ $\quad -3 \le x \le 3$ uppfyller ekv.

$\Rightarrow$ $\quad 0 < x < 3$ uppfyller ekv.

$\therefore$ Svar: $\underline{\underline{A}}$

16.

$$\frac{\frac{1}{7}+\frac{1}{5}}{\frac{2}{15}-\frac{7}{12}} = \frac{\frac{5+7}{35}}{\frac{12\cdot2-7\cdot15}{15\cdot12}} = \frac{12\cdot\overset{3}{\cancel{15}}\cdot12}{\underset{7}{\cancel{35}}\cdot(24-105)}$$

$$= \frac{\overset{4}{\cancel{12}}\cdot\overset{1}{\cancel{3}}\cdot\overset{4}{\cancel{12}}}{-\underset{\underset{\underset{3}{\cancel{9}}}{\cancel{27}}}{\cancel{81}}\cdot7} = \underline{\underline{-\frac{16}{21}}}$$

17.

$$\log_{5\sqrt{5}} 5 = \log_{5^{3/2}} 5 = \log_{5^{3/2}} \left(5^{3/2}\right)^{2/3}$$

$$= \frac{2}{3}\log_{5^{3/2}} 5^{3/2} = \frac{2}{3}\cdot 1 = \underline{\underline{2/3}}$$

18.

$$S = 2 + a_2 + a_3 + 54$$

$$= 2 + 2c + 2c^2 + \underbrace{54}_{= 2c^3}$$

$$2c^3 = 54$$

$$c = \sqrt[3]{54/2} = \sqrt[3]{27} = 3$$

$$\Rightarrow \quad S = 2 + 2 \cdot 3 + 2 \cdot 9 + \underbrace{54}_{= 2 \cdot 27} =$$

$$= 2 + 6 + 18 + 54 = \underline{\underline{80}}$$

19.

$$f(x) = \cos^2 x + \sin x - 3$$

$$f'(x) = 2\cos x \cdot (-\sin x) + \cos x$$

$$= -2\sin x \cos x + \cos x$$

$$= -\sin 2x + \cos x$$

$$f'(x) = 0 \Leftrightarrow \quad \sin 2x = \cos x$$

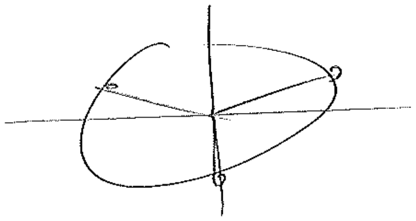
$$2x = \pi/2 - x + 2\pi n$$

$$3x = \pi/2 + 2\pi n$$

$$x = \pi/6 + \frac{2\pi}{3} n$$

$$= \pi/6 , \pi/6 + \frac{2\pi}{3} , \pi/6 + \frac{4\pi}{3} , \ldots$$

$$= \frac{\pi}{6} , \frac{5\pi}{6} , \underbrace{\frac{9\pi}{6}}_{= \frac{3\pi}{2}} , \ldots$$



$f(\pi/6) = (\sqrt{3}/2)^2 + \frac{1}{2} - 3$

$= 3/4 + 1/2 - 3$

$= 3/4 + 2/4 - 12/4$

$= -7/4$

$f(5\pi/6) = (-\sqrt{3}/2)^2 + \frac{1}{2} - 3 = -7/4$

$f(3\pi/2) = 0 - 1 - 3 = -4$

$\therefore$ Svar: $\underline{\underline{-4}}$

20.

$$2^{\pi - x}\cos(\pi x) = (-1)^{x+10}\cdot 4^{x+\frac{\pi}{2}-3}$$

$$\cos(\pi x) = (-1)^{x+10}\cdot (2^2)^{x+\frac{\pi}{2}-3}\cdot 2^{x-\pi}$$

$$\cos(\pi x) = (-1)^{x+10}\cdot 2^{2x+\cancel{\pi}-6}\cdot 2^{x-\cancel{\pi}}$$

$$\underbrace{\cos(\pi x)}_{VL} = \underbrace{(-1)^{x+10}\cdot 2^{3x-6}}_{HL}$$

$|VL| \le 1$

$|HL| = 2^{3x-6} > 1$ för $3x - 6 > 0$

$\Leftrightarrow 3x > 6$

$\Leftrightarrow x > 2$

$\Rightarrow$ Lösning saknas för $x > 2$

$VL(2) = \cos(2\pi) = 1$

$HL(2) = (-1)^{12} \cdot 2^{3 \cdot 2 - 6} = 2^0 = 1$

$\therefore$ $x = 2$ är en heltalslösning och därmed största heltalslösning.

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c.

$\sin(\ln x) = \cos(\ln x^2)$

$\sin(\ln x) = \sin(\pi/2 - 2\ln x)$

$\ln x = \pi/2 - 2\ln x + 2\pi n$

$3\ln x = \pi/2 + 2\pi n$

$$\underline{\underline{x = e^{\pi/6 + 2\pi/3\, n}}}$$
