LÖSNINGAR TENTA 150112
LLMA60 / MMGL61 Analys

1. Lös $|x+6| - |2x-4| = 1$.

För $x > 2$: $x+6 - (2x-4) = 1$

$9 = x$

$x \leq -6$ $\quad -x-6+2x-4 = 1$

$x = 11 > -6$ ↯

För $-6 < x < 2$:

$x+6+2x-4 = 1$

$3x = -1 \qquad x = -\frac{1}{3}$

Svar: $-\frac{1}{3}, 9$

2. a) $\int x \sin x^2 \, dx \underset{\substack{x^2 = u \\ 2x\,dx = du}}{=} \frac{1}{2}\int \sin u \, du =$

$= -\frac{1}{2}\cos u + C = -\frac{1}{2}\cos x^2 + C$

b) $\int x^2 \sin x \, dx \underset{PI}{=} -x^2 \cos x + \int 2x \cos x \, dx \underset{PI}{=}$

$= -x^2\cos x + 2x\sin x - 2\int \sin x \, dx =$

$= -x^2 \cos x + 2x \sin x + 2\cos x + C.$

3. $f(x) = x^3 e^{-x}$

$f'(x) = (3x^2 - x^3)e^{-x}$

$f''(x) = (6x - 6x^2 + x^3)e^{-x}$

Teckenstudie:

x		0		3	
f	↗		↗	Max	↘
f'	$+$	0	$+$	0	$-$

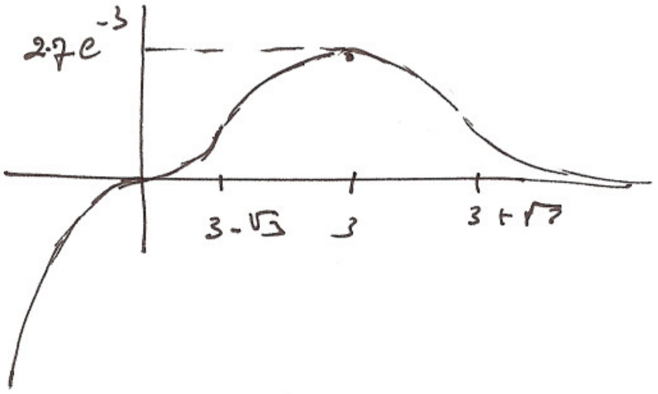
Vi har $\lim_{x \to \infty} f(x) = 0$, $\lim_{x \to -\infty} f(x) = -\infty$

$f'(x) = 0$: $x = 0$ eller $x = 3$

$f(0) = 0$, $f(3) = 27e^{-3}$

Inflexionspunkter: $f''(x) = 0$

$x = 0$ eller $x^2 - 6x + 6 = 0$ $\quad x = 3 \pm \sqrt{3}$



4. $$y = \frac{x^2}{4} - \ln\sqrt{x} = \frac{x^2}{4} - \frac{1}{2}\ln x$$

Längd: $\int_1^4 \sqrt{1+(y')^2}\,dx$

$$y' = \frac{x}{2} - \frac{1}{2x} \qquad \sqrt{1+y'^2} = \sqrt{1+\left(\frac{x}{2}-\frac{1}{2x}\right)^2} =$$

$$= \sqrt{1+\frac{x^2}{4} - \frac{1}{2} + \frac{1}{4x^2}} = \sqrt{\frac{x^2}{4} + \frac{1}{2} + \frac{1}{4x^2}} = \sqrt{\left(\frac{x}{2} + \frac{1}{2x}\right)^2} = \frac{x}{2} + \frac{1}{2x}$$

Längd:

$$\int_1^4 \frac{x}{2} + \frac{1}{2x}\,dx = \frac{x^2}{4} + \ln\sqrt{x}\,\Big|_1^4 =$$

$$\frac{16}{4} + \ln 2 - \frac{1}{4} = \frac{15}{4} + \ln 2.$$

5. $x y' - y = x^2 \sin x$

$y' - \frac{y}{x} = x \sin x$

IF $e^{-\int \frac{dx}{x}} = \frac{1}{x}$

$\frac{y'}{x} - \frac{y}{x^2} = \left(\frac{y}{x}\right)' = \sin x$

$\frac{y}{x} = \int \sin x \, dx = -\cos x + C$

$y = -x \cos x + C x$

$y\left(\frac{\pi}{2}\right) = 1 \; ; \quad C \frac{\pi}{2} = 1 , \quad C = \frac{2}{\pi}$

Svar: $y = -x \cos x + \frac{2x}{\pi}$

6. $f = x^3 y - x y + x y^3$

$\frac{\partial f}{\partial x} = 3x^2 y - y + y^3$

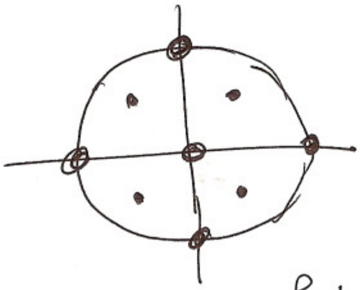
$\frac{\partial f}{\partial y} = x^3 - x + 3 x y^2$

$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0 \; : \quad \begin{cases} (3x^2 - 1 + y^2) y = 0 \\ (x^2 - 1 + 3y^2) x = 0 \end{cases}$

$\begin{cases} y = 0 \\ x (x^2 - 1) = 0 \end{cases}$ eller $\begin{cases} y^2 = 1 \\ x = 0 \end{cases}$ eller

$\begin{cases} 3x^2 - 1 + y^2 = 0 \\ x^2 - 1 + 3y^2 = 0 \end{cases} \Leftrightarrow \begin{cases} 2x^2 - 2y^2 = 0 \\ 4x^2 - 1 = 0 \end{cases}$

Lösningar: $(0,0)$, $(\pm 1, 0)$, $(0, \pm 1)$, $(\mp \frac{1}{2}, \pm \frac{1}{2})$



$f=0 \iff xy(x^2+y^2-1)=0$

$(0,0)$ och $(\pm 1,0)$, $(0,\pm 1)$

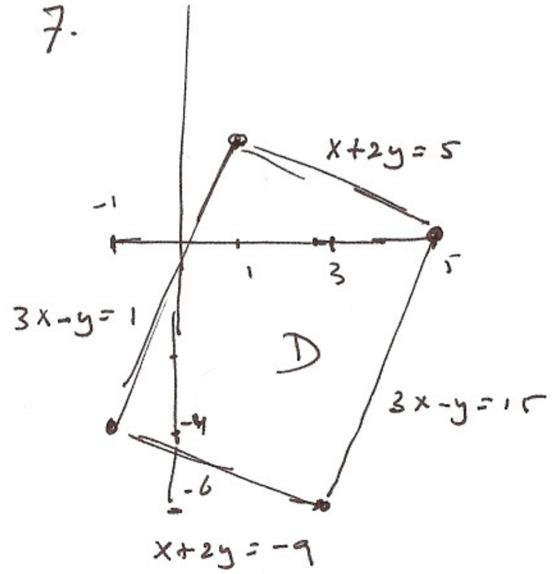
sadelpunkter.

$f(\frac{1}{2},\frac{1}{2}) = f(-\frac{1}{2},-\frac{1}{2}) = \frac{1}{4}(\frac{1}{2}-1) = -\frac{1}{8}$

$f(\frac{1}{2},-\frac{1}{2}) = f(-\frac{1}{2},\frac{1}{2}) = \frac{1}{8}$

$(\frac{1}{2},\frac{1}{2})$ och $(-\frac{1}{2},-\frac{1}{2})$ min , $(\frac{1}{2},-\frac{1}{2}),(-\frac{1}{2},\frac{1}{2})$ max.

7.



$u = 3x-y$

$v = x+2y$

$du\,dv = \begin{vmatrix} 3 & -1 \\ 1 & 2 \end{vmatrix} dx\,dy = 7\,dx\,dy$

$\iint_D (3x-y)\cos(x+2y)\,dA =$

$\frac{1}{7}\int_{-9}^{5}\int_{1}^{15} u\cos v\,du\,dv =$

$\frac{1}{7}\left(\frac{1}{2}u^2\Big|_1^{15}\right)\cdot\left(\sin v\Big|_{-9}^{5}\right) =$

$\frac{112}{7}\left(\sin 5 - \sin(-9)\right) = 16(\sin 5 + \sin 9)$

8. $\int_0^{2015} x(x-2015)^{2014}\,dx \underset{PI}{=}$

$\underbrace{\frac{1}{2015}x(x-2015)^{2015}\Big|_0^{2015}}_{=0} - \frac{1}{2015}\int_0^{2015}(x-2015)^{2015}\,dx =$

$-\frac{1}{2015\cdot 2016}(x-2015)^{2016}\Big|_0^{2015} =$

$0 + \frac{1}{2015\cdot 2016}(-2015)^{2016} = \frac{(2015)^{2015}}{2016}.$

JS