INTEGRATIONSTEORI (5p)
(INTEGRATION THEORY)
(GU$[MAF440]$ **,CTH**$[TMV100]$**)**

ASSIGNMENT 3
(Must be handed in before Friday at 9 am, week 51)
Note that 1 dp=0.1p.

1. (1dp) Suppose $f \in L^1(m)$ and $F(x) = \int_{-\infty}^{x} f(t)dt$, $-\infty < x < \infty$. Prove that F is continuous.

2. (2dp) Suppose $f \in L^1(m)$ and $g(x) = \frac{1}{2}\int_{x-1}^{x+1} f(t)dt$, $x \in \mathbf{R}$. Prove that

$$\int_{\mathbf{R}} \mid g(x) \mid dx \leq \int_{\mathbf{R}} \mid f(x) \mid dx.$$

3. (2dp) For $t > 0$ and $x \in \mathbf{R}$ let

$$g(t,x) = \frac{1}{\sqrt{2\pi t}} e^{-\frac{x^2}{2t}}$$

and

$$h(t,x) = \frac{\partial g}{\partial t}.$$

Given $a > 0$, prove that

$$\int_{-\infty}^{\infty} (\int_{a}^{\infty} h(t,x)dt)dx = -1$$

and

$$\int_{a}^{\infty} (\int_{-\infty}^{\infty} h(t,x)dx)dt = 0$$

and conclude that

$$\int_{[a,\infty[\times \mathbf{R}} \mid h(t,x) \mid dtdx = \infty.$$

(Hint: First prove that

$$\int_{-\infty}^{\infty} g(t,x)dx = 1$$

and

$$\frac{\partial g}{\partial t} = \frac{1}{2}\frac{\partial^2 g}{\partial x^2}.)$$