MAF620 Functional Analysis 2001/2002
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Exercises — 1

Assume that $\mathbf{R}$ is equipped with the standard topology if nothing else is mentioned.

1. Let $(X, \mathcal{T}_X)$ and $(Y, \mathcal{T}_Y)$ be topological spaces. Show that the following statements are equivalent:

   (a) $f : X \to Y$ is continuous;

   (b) $f(\overline{A}) \subset \overline{f(A)}$ for all $A \subset X$;

   (c) $\overline{f^{-1}(B)} \subset f^{-1}(\overline{B})$ for all $B \subset Y$.

2. Let $(X, \mathcal{T}_X)$ be a normal space. Assume $A \subset X$ is a closed G_δ-set, i.e. A is a countable intersection of open sets in X. Show that there is a continuous real-valued function $f \in C(X)$ such that $f(x) = 0$ iff $x \in A$.

3. Let (X, d) be a metric space. Show that every open set is a F_σ-set, i.e. a countable union of closed set, and that every closed set is a G_δ-set.

4. Let $(X_\alpha, \mathcal{T}_\alpha)$, $\alpha \in A$, be a collection of topological spaces, and let the Cartesian product $\Pi_{\alpha \in A} X_\alpha$ be equipped with the topology given by the base $\mathcal{B} = \{\Pi_{\alpha \in A} U_\alpha : U_\alpha \in \mathcal{T}_\alpha\}$. Verify that $\mathcal{B}$ is a base for a topology. This topology is called the **box-topology** and coincides with the product topology for finite index sets A. Why is the product topology "to prefer"?

Hand in your solutions at the latest on the **4/3**.