MAF620 Functional Analysis 2001/2002
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Exercises — 2

Assume that $\mathbf{R}$ is equipped with the standard topology if nothing else is mentioned.

1. Prove[1] that $\{0,1\}^{\mathbf{N}}$ with the product topology is homeomorphic to the Cantor set C, see Folland page 38.

2. Let X_α, $\alpha \in A$ be a family of topological spaces of which infinitely many are noncompact. Show that every closed compact subset of $\Pi_{\alpha \in A} X_\alpha$ is nowhere dense.

3. Let X be a compact Hausdorff space and let f_n, $n = 1, 2, \ldots$, be a sequence of continuous real-valued on X such that

   $$f_n(x) \geq f_{n+1}(x)$$

   for all $n \in \mathbf{N}$. Let $f_n(x) \to f(x)$ pointwise. Show that f_n converges uniformly to f iff f is continuous.

4. Let h is a continuous, strictly monotone real-valued function defined on $[0,1]$. Assume that f is a continuous real-valued function on $[0,1]$ such that

   $$\int_0^1 h^n(x) f(x) dx = 0, \quad n = 0, 1, 2, \ldots$$

   Prove that $f = 0$.

Hand in your solutions at the latest on the **18/3**.

[1] Hint: consider the mapping $\{0,1\}^{\mathbf{N}} \ni 9x_1, x_2, \ldots) \mapsto \Sigma_{n=1}^{\infty} 2x_n/3^n \in C$.