MAF620 Functional Analysis 2001/2002
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Exercises — 3

1. Let X be an infinite-dimensional Banach space. Prove that that there exists a linear functional on X that is not continuous.

2. Let X be a Banach space and let C be a convex set. Show that C is closed in the norm topology iff it is weakly closed.

3. Let $A = (a_{i,j})$, $i, j = 1, 2, \ldots$, be a matrix of complex numbers that is row finite, i.e. $a_{i,j} = 0$ for large j when i is fixed. For every sequence of complex numbers $s_1, s_2, \ldots$ set

$$(As)_i = \Sigma_j a_{i,j} s_j, \quad i = 1, 2, \ldots.$$

   We say that the sequence s_i is summable with the method (A) if $(As)_i$ has a limit when $i \to \infty$. Prove that every convergent sequence is summable with the usual limit with the method (A) iff for some constant M

$$\Sigma_j |a_{i,j}| \leq M, \quad i = 1, 2, \ldots$$

$$\lim_{i \to \infty} a_{i,j} = 0$$

   for fixed j and

$$\lim_{i \to \infty} \Sigma_j a_{i,j} = 1.$$

4. Prove the mean ergodic theorem on page 178 in Folland (exercise 67).

Hand in your solutions at the latest on the **18/4**.