MAF620 Functional Analysis 2001/2002
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Exercises — 4

1. Let T be a bounded linear mapping from a Banach space X into a Banach space Y. Assume that the range
$$\mathcal{R}(T) = \{Tx : x \in X\}$$
has finite codimension. Show that $\mathcal{R}(T)$ is closed.

2. Prove that
   (a) if $\Sigma_{k=1}^{\infty} a_k x_k$ converges for all $a = (a_k) \in l^1$, then $x = (x_k) \in l^\infty$.
   (b) $\Sigma_{k=1}^{\infty} a_k x_k$ converges for all $a = (a_k) \in c$, where $c \subset l^\infty$ is the subspace of all sequences with finite limit, then $x = (x_k) \in l^1$.

3. Show that $L^p(\mu)$ is strictly convex for $1 < p < \infty$, i.e. show that
$$\|f\|_p = \|g\|_p = 1, \;\; f \neq g \implies \|\frac{1}{2}(f+g)\|_p < 1.$$

4. Show that a sequence $\{f_n\}_{n=1}^{\infty}$ converges weakly to $f \in C_0(X)$ iff $\sup_n \|f_n\|_u < \infty$ and $f_n \to f$ pointwise.

Hand in your solutions at the latest on the **15/5**.