MAI520 Beräkningsgeometri 2001/2002
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Exercises — 1

1. Prove that every polygon with a hole can be triangulated. What can be said about the number of triangles?
2. Give a (simple) polygon together with a distribution of guards that cover all parts of the walls (boundary of the polygon) but not the whole of the interior.
3. Let P be a (simple) polygon. Give an algorithm that in $O(n \log n)$ time finds a diagonal of P that splits the polygon into two (simple) polygons with at most $\lceil \frac{3n}{4} \rceil$ vertices each[1].
4. Let $k \geq 1$, $l > 1$ and $T : \mathbf{Z}_+ \to \mathbf{R}_+$ be given, where $T(n)$ satisfies the recursion formula
$$T(n) = kT([\frac{n}{l}]) + f(n), \quad n = 1, 2, \ldots$$
for some function $f : \mathbf{Z}_+ \to \mathbf{R}_+$. Show that
   (a) $T(n) = \Theta(n^{{}^l\log k})$ if $f(n) = O(n^{{}^l\log k - \epsilon})$ for some $\epsilon > 0$.
   (b) $T(n) = \Theta(n^{{}^l\log k} \log n)$ if $f(n) = \Theta(n^{{}^l\log k})$.
5. Suppose you are given two polygonal chains, A and B, each represented as a sequence of points in the plane. Suppose further that the sequence of points in both A and B are ordered by increasing x–coordinates. Describe an efficient method for determining if the chains A and B ever cross each other. What is the running time of your method?

Hand in your solutions at the latest on the **24/9**.

[1] Hint: Use the dual graph of the triangulation