

$$P = (1, 1, 3) \quad Q = (1, 0, -1) \quad R = (4, 3, 2)$$

$$\begin{aligned} \vec{u} &= \vec{PQ} = (1, 0, -1) - (1, 1, 3) = (0, -1, -4) \\ \vec{v} &= \vec{PR} = (4, 3, 2) - (1, 1, 3) = (3, 2, -1) \end{aligned}$$

Dessär är parallella med planet, så

$$\vec{n} = \vec{u} \times \vec{v} \text{ är en normalvektor till planet.}$$

$$\vec{n} = \left( \begin{vmatrix} -1 & -4 \\ 2 & -1 \end{vmatrix}, -\begin{vmatrix} 0 & -4 \\ 3 & -1 \end{vmatrix}, \begin{vmatrix} 0 & -1 \\ 3 & 2 \end{vmatrix} \right) = (9, -12, 3)$$

$(x, y, z)$  ligger i planet  $\Leftrightarrow$

$$(9, -12, 3) \cdot (x^{-1}, y^0, z^{(-1)}) = 0$$

$\Leftrightarrow$

$$9(x^{-1}) - 12y^0 + 3(z^{(-1)}) = 0$$

$\Leftrightarrow$

$$3x^{-1} - 4y^0 + z^{(-1)} = 2$$

$\Leftrightarrow$

Svar Planet ekvation är  $3x^{-1} - 4y^0 + z^{(-1)} = 2$ .

7.

$$\begin{aligned} \text{Homogen ekvation: } y_u'' + 4y_u &= 0 \\ \text{Karakteristisk ekvation: } \lambda^2 + 4 &= 0 \\ \lambda &= \pm 2i \end{aligned}$$

$$\text{Så } y_u(t) = A \sin(2t) + B \cos(2t)$$

Aussätt partiellär lösnings:

$$y_p = C s \int u t + D c \int s t$$

$$y_p' = C \cos t - D \sin t$$

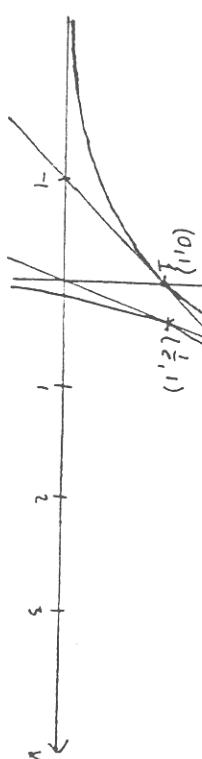
$$y_p'' = -C s \int u t - D c \int s t$$

$$\begin{aligned} y_p'' + 4y_p &= -C s \int u t - D c \int s t + 4(C s \int u t + D c \int s t) \\ &= 3C s \int u t + 3D c \int s t = C \cos t \end{aligned}$$

vilka

$$\begin{cases} 3C = 0 \\ 3D = 1 \end{cases} \Rightarrow \begin{cases} C = 0 \\ D = \frac{1}{3} \end{cases}$$

$$y_p = \frac{1}{3} \cdot \cos t$$



8)

Svar

$$y = e^x = f(x) \quad f'(x) = e^x$$

Tangent till  $y = f(x)$  i punkten  $(a, f(a))$ :

$$y - f(a) = f'(a)(x-a) \quad (\Rightarrow)$$

$$y - e^a = e^a(x-a) \quad (\Rightarrow)$$

$$y = e^a x + e^a(1-a)$$

$$y = 2 + \ln x = g(x) \quad g'(x) = \frac{1}{x}$$

Tangent till  $y = g(x)$  i punkten  $(b, g(b))$ :

$$y - g(b) = g'(b)(x-b) \quad (\Rightarrow)$$

$$y - (2 + \ln b) = \frac{1}{b}(x-b) \quad (\Rightarrow)$$

$$y = \frac{1}{b}x + 1 + \ln b$$

$$\text{Generala mera tangenter får av } \begin{cases} e^a = \frac{1}{b} \\ e^a(1-a) = 1 + \ln b \end{cases} \quad (1)$$

$$(1) \text{ ger } b = e^{-a}, \text{ stoppa in i: (2): } e^a(1-a) = 1 - a \quad (\Rightarrow)$$

$a=0$  eller  $a=1$ . Alltså, två lösningar  $\begin{cases} a=0 \\ b=1 \end{cases}$  respektive  $\begin{cases} a=1 \\ b=\frac{1}{e} \end{cases}$

som ger tangenterna  $y = x + 1$  resp.  $y = e^x$ .

$$\begin{array}{l} y = e^x \\ y = f(x) \\ y = x + 1 \end{array}$$

$$y = 2 + \ln x$$

$$\begin{cases} 3C = 0 \\ 3D = 1 \end{cases} \Rightarrow \begin{cases} C = 0 \\ D = \frac{1}{3} \end{cases}$$

$$y_p = \frac{1}{3} \cdot \cos t$$

Akkuraten (lösning till ekvationen

$$y = y_p + y_n = \frac{1}{3} \cos t + A s \int u t + B \cos(2t), A, B \in \mathbb{R}.$$

$$\begin{aligned} y' &= -\frac{1}{3} s \int u t + 2A \cos(2t) - 2B s \int u t \\ y(0) &= 0 \quad \text{gör} \quad \frac{1}{3} + B = 0 \quad \text{dvs} \quad B = -\frac{1}{3} \\ y'(0) &= 2 \quad \text{gör} \quad 2A = 2 \quad \text{dvs} \quad A = 1 \end{aligned}$$