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Naturvetarmatematik A, MAN110

Lösningar

Håkan Grannath

1) $\sqrt{x^2+1} + 2x+1=0 \Leftrightarrow \sqrt{x^2+1} = -2x-1 \stackrel{(*)}{\Rightarrow}$

$\Rightarrow x^2+1 = (2x+1)^2 \Leftrightarrow 0 = 3x^2+4x \Leftrightarrow x(3x+4)=0$

så $x=0$ eller $x=-4/3$. Pga. (*) måste lösningarna prövas, varvid $x=0$ förkastas.

Svar $x = -4/3$.

2) $\frac{x-4}{x+1} - \frac{1}{x} \leq 1 \Leftrightarrow \frac{x-4}{x+1} - \frac{1}{x} - 1 \leq 0 \Leftrightarrow$

$\Leftrightarrow \frac{x(x-4) - (x+1) - x(x+1)}{x(x+1)} \leq 0 \Leftrightarrow \frac{-6x-1}{x(x+1)} \leq 0 \Leftrightarrow$

$\Leftrightarrow \frac{6x+1}{x(x+1)} \geq 0$.

x	-1	-1/6	0
6x+1	-	-	0
x	-	-	0
x+1	-	0	+
6x+1	-	+	0
x(x+1)	-	+	+

Svar: $x > 0$ eller $-1 < x \leq -\frac{1}{6}$

3) a) $\lim_{x \rightarrow 0} \frac{\ln(1-\sin^2 x)}{x^2} = \frac{0}{0} = [\text{l'Hospital}] =$

$= \lim_{x \rightarrow 0} \frac{\frac{1}{1-\sin^2 x} (-2 \sin x \cos x)}{2x} = \lim_{x \rightarrow 0} -\frac{\cos x}{1-\sin^2 x} \cdot \frac{\sin x}{x} = -1$

ty $\lim_{x \rightarrow 0} \frac{\sin x}{x} = \frac{0}{0} = [\text{l'Hospital}] =$

$= \lim_{x \rightarrow 0} \frac{\cos x}{1} = \frac{1}{1} = 1$

Svar: -1