(3)

Naturvetarmatematik A, MAN110 2/1-2002

Håkan Granath

Lösningar

1) $\sqrt{3x+3} = 3x+1 \overset{(*)}{\Longrightarrow} 3x+3 = (3x+1)^2 \Leftrightarrow 9x^2+3x-2=0$

$\Leftrightarrow \quad (3x+\frac{1}{2}) = 2+\frac{1}{4} \quad \Leftrightarrow \quad 3x+\frac{1}{2} = \pm\frac{3}{2}$

$\Leftrightarrow \quad x = \frac{-1\pm 3}{6}$ så $x_1 = -\frac{2}{3}$ $\quad x_2 = 1/3$

Pga. (*) måste vi prova lösningarna:

$x = -\frac{2}{3}$ ger $\sqrt{1} = -1$ falskt

$x = 1/3$ ger $\sqrt{4} = 2$ sant

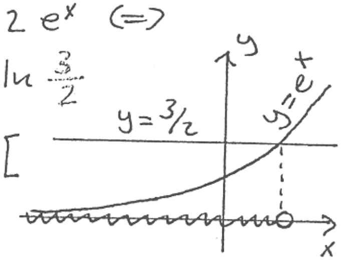
Svar $x = 1/3$.

2) $f(x)$ är definierad $\Leftrightarrow 3e^{2x} - 2e^{3x} > 0 \Leftrightarrow$

$\Leftrightarrow \quad 3e^{2x} > 2e^{3x} \quad \Leftrightarrow \quad 3 > 2e^x \quad \Leftrightarrow$

$\Leftrightarrow \quad e^x < \frac{3}{2} \quad \Leftrightarrow \quad x < \ln\frac{3}{2}$

Svar $D_f =]-\infty, \ln\frac{3}{2}[$



3) a) $\lim_{x\to 0} \frac{\ln(\overset{\to 1}{\widetilde{\cos x}})}{e^x - e^{-x} - 2} = \text{"}\frac{0}{0}\text{"} = [\text{l'Hospital}] =$

$= \lim_{x\to 0} \frac{\frac{1}{\cos x}(-\sin x)}{e^x - e^{-x}} = \lim_{x\to 0} \frac{-\tan x}{e^x - e^{-x}} = \text{"}\frac{0}{0}\text{"} = [\text{l'Hosp.}] =$

$= \lim_{x\to 0} \frac{-\frac{1}{\cos^2 x}}{e^x + e^{-x}} = \frac{-\frac{1}{1}}{1+1} = \underline{\underline{-\frac{1}{2}}}$

b) $\lim_{x\to\infty} \frac{x^{10} + 3e^{2x}}{2\ln x + e^{2x}} = \lim_{x\to\infty} \frac{x^{10}/e^{2x} + 3}{2\ln x/e^{2x} + 1} =$

$= \frac{0+3}{0+1} = \underline{\underline{3}}$, ty $\quad x^{10} << e^{2x}$

och $\ln x << e^{2x}$ då $x \to \infty$

4) $f(x) = \frac{1}{2}\ln(x^2+1) - 2\arctan x + x^2 - 4x$

$f'(x) = \frac{1}{2}\frac{1}{x^2+1}2x - 2\frac{1}{x^2+1} + 2x - 4 =$

$= \frac{x - 2 + (2x-4)x^2+1}{x^2+1} = \frac{2x^3-4x^2+3x-6}{x^2+1}$

$$\begin{array}{r|l} & 2x^2 \quad +3 \\ \hline x-2 & 2x^3-4x^2+3x-6 \\ & -(2x^3-4x^2) \\ \hline & 3x-6 \\ & -(3x-6) \\ \hline & 0 \end{array}$$

så $f'(x) = (x-2)\underbrace{\frac{2x^2+3}{x^2+1}}_{>0}$

$f'(x) = 0 \Leftrightarrow x = 2$

x		2	
$f'(x)$	$-$	0	$+$
$f(x)$	↘	$f(2)$	↗

Svar $f_{min} = f(2) = \frac{1}{2}\ln 5 - 2\arctan 2 - 4$.

5) $\int_0^1 e^{x^{1/3}} dx = \left[\begin{matrix} t = x^{1/3} & dx = 3t^2 dt \\ x = t^3 & \end{matrix} \quad \begin{array}{c|c} x & t \\ \hline 0 & 0 \\ 1 & 1 \end{array}\right] =$

$= \int_0^1 e^t \cdot 3t^2 dt \overset{\text{P.I.}}{=} \left[e^t 3t^2\right]_0^1 - \int_0^1 e^t 6t\,dt =$

$= (3e-0) - 6\int_0^1 t e^t dt \overset{\text{P.I.}}{=} 3e - 6\left(\left[te^t\right]_0^1 - \int_0^1 e^t dt\right) =$

$= 3e - 6\left((e-0) - \left[e^t\right]_0^1\right) = \underline{\underline{3e-6}}$

6) a) $\begin{vmatrix} 1-\lambda & -2 \\ 1 & 4-\lambda \end{vmatrix} = 0 \quad \Leftrightarrow \quad (1-\lambda)(4-\lambda) - 1(-2) = 0$

$\Leftrightarrow \quad \lambda^2 - 5\lambda + 6 = 0 \quad \Leftrightarrow \quad (\lambda-2)(\lambda-3) = 0$

så $\lambda_1 = 2$, $\lambda_2 = 3$

Egenvektoren $f_1 = \begin{bmatrix} x \\ y \end{bmatrix}$ med egenvärde $\lambda_1 = 2$:

$\begin{cases} (1-2)x - 2y = 0 \\ 1\cdot x + (4-2)y = 0 \end{cases} \Leftrightarrow \quad x + 2y = 0 \Leftrightarrow \begin{cases} x = -2t \\ y = t \end{cases} \quad t \in \mathbb{R}$