Naturvetarmatematik A; 1ANII 11/10-2000

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Lösningar

1) Brytpunkterna är $x=0$ och $x=3$.

om $x \geq 3$: $2x+(3-x)=1 \iff x=-2$ men detta uppfyller ej $x \geq 3$.

om $0 \leq x < 3$: $2x-(3-x)=1 \iff x = 4/3$

om $x<0$: $-2x-(3-x)=1 \iff x=-4$

Svar $x = 4/3$ eller $x=-4$.

2) $$\frac{3}{x+1} + \frac{x+3}{x^2+1} \leq \frac{4}{x} \iff \frac{3}{x+1} + \frac{x+3}{x^2+1} - \frac{4}{x} \leq 0 \iff$$

$$\frac{3x(x^2+1)+(x+3)x(x+1)-4(x+1)(x^2+1)}{x(x+1)(x^2+1)} \leq 0 \iff$$

$$\frac{2(x-2)}{x(x+1)(x^2+1)} \leq 0$$

x				-1			0			2		
$x-2$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	0	$+$	$+$
x	$-$	$-$	$-$	$-$	$-$	$-$	0	$+$	$+$	$+$	$+$	$+$
$x+1$	$-$	$-$	$-$	0	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
$\frac{2(x-2)}{x(x+1)\underbrace{(x^2+1)}_{>0}}$	$-$	$-$	$-$	ξ	$+$	$+$	ξ	$-$	$-$	0	$+$	$+$

Svar $x<-1$ eller $0<x\leq 2$.

3) $$\lim_{x\to 0} \frac{\ln(\cos x)}{e^{-x}+\sin x-\cos x} = \text{"}\frac{\ln 1}{1+0-1} = \frac{0}{0}\text{"} = [\text{l'Hospital}] =$$

$$= \lim_{x\to 0} \frac{-\frac{\sin x}{\cos x}}{-e^{-x}+\cos x+\sin x} = \text{"}\frac{0}{0}\text{"} = [\text{l'Hospital}] =$$

$$= \lim_{x\to 0} \frac{-\frac{1}{\cos^2 x}}{e^{-x}-\sin x+\cos x} = \frac{-1}{1-0+1} = \underline{\underline{-\frac{1}{2}}}$$

4) $$f(x) = \frac{x^2+x}{3x^2+1} \qquad f'(x) = \frac{(3x^2+1)(2x+1)-(x^2+x)6x}{(3x^2+1)^2}$$

$$= \frac{-3x^2+2x+1}{(3x^2+1)^2} = \frac{-(x-1)(3x+1)}{(3x^2+1)^2}$$

så $f'(x)=0 \iff x=1$ eller $x=-\frac{1}{3}$.

x	$(-\infty)$		$-\frac{1}{3}$		1		(∞)
f'		$-$	0	$+$	0	$-$	
f	$(\frac{1}{3})$	$\searrow$	$-\frac{1}{6}$	$\nearrow$	$\frac{1}{2}$	$\searrow$	$(\frac{1}{3})$

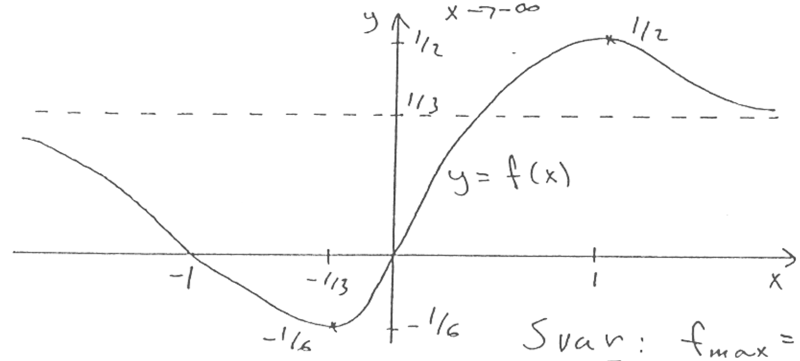
$f'(0)=1>0$

$f(1) = \frac{2}{4} = \frac{1}{2}$

$f(-\frac{1}{3}) = \frac{1/9-1/3}{1/3+1} = -1/6$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{1+1/x}{3+1/x^2} = 1/3$$

och p.s.s. är $\lim_{x\to-\infty} f(x) = \frac{1}{3}$.



Svar: $f_{max} = f(1) = 1/2$

$f_{min} = f(-\frac{1}{3}) = -1/6$

5) a) $$\int_0^1 \frac{1+x}{1+\sqrt{x}}\,dx = \left[\begin{array}{l|l} t=\sqrt{x} & \begin{array}{c|c} x & t \\ \hline 0 & 0 \\ 1 & 1 \end{array} \\ x=t^2 & \\ dx = 2t\,dt & \end{array}\right] =$$

$$= \int_0^1 \frac{1+t^2}{1+t}\cdot 2t\,dt = 2\int_0^1 \frac{t^3+t}{t+1}\,dt =$$

$$\begin{array}{r|l} & t^2-t+2 \\ \hline t+1 & t^3+t \\ & -(t^3+t^2) \\ \hline & -t^2+t \\ & -(-t^2-t) \\ \hline & 2t \\ & -(2t+2) \\ \hline & -2 \end{array}$$

$$= 2\int_0^1 t^2-t+2-\frac{2}{t+1}\,dt =$$

$$= 2\left[t^3/3 - t^2/2 + 2t - 2\ln|t+1|\right]_0^1 =$$

$$= 2\left(\left(\frac{1}{3}-\frac{1}{2}+2-2\ln 2\right)-0\right) = \underline{\underline{\frac{11}{3}-4\ln 2}}$$