Naturvetarmatematik II, MAN110 26/8-2000

## Lösningar

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1) $\sqrt{2-x} = 1-2x \overset{(*)}{\Rightarrow} 2-x = (1-2x)^2 \Leftrightarrow$

$\Leftrightarrow 2-x = 1-4x+4x^2 \Leftrightarrow 4x^2-3x-1 = 0 \Leftrightarrow$

$\Leftrightarrow (x-1)(4x+1) = 0 \Leftrightarrow x=1$ eller $x=-1/4$

P.g.a. (*) måste vi pröva lösningarna:

$x=1$ ger $\sqrt{1} = -1$ falskt

$x=-1/4$ ger $\sqrt{9/4} = 3/2$ sant

Svar: $x = -1/4$.

2) $\frac{2}{x} \le \frac{1}{x-1} \Leftrightarrow \frac{2}{x} - \frac{1}{x-1} \le 0 \Leftrightarrow$

$\Leftrightarrow \frac{2(x-1)-x}{x(x-1)} \le 0 \Leftrightarrow \frac{x-2}{x(x-1)} \le 0$

Teckentabell:

| $x$ | | | 0 | | | 1 | | 2 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $x-2$ | − | − | − | − | − | − | − | 0 | + | + |
| $x$ | − | − | 0 | + | + | + | + | + | + | + |
| $x-1$ | − | − | − | − | − | 0 | + | + | + | + |
| $\frac{x-2}{x(x-1)}$ | − | − | ‖ | + | + | ‖ | − | 0 | + | + |

Svar $x<0$ eller $1<x\le 2$.

3) $\lim_{x\to 0} \frac{e^{x^2}+\ln(1+2x)-1}{x\cos(3x)} = \text{"}\frac{1+0-1}{0\cdot 1} = \frac{0}{0}\text{"} = [\text{l'Hospital}] =$

$= \lim_{x\to 0} \frac{2xe^{x^2}+\frac{1}{1+2x}\cdot 2}{\cos(3x)-3x\sin(3x)} = \frac{0+1\cdot 2}{1-0} = \underline{\underline{2}}$

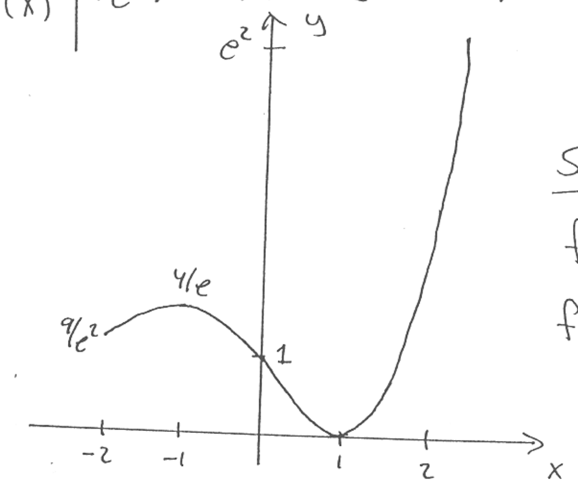
4) $f(x) = (x-1)^2 e^x \qquad x \in [-2,2]$

$f'(x) = 2(x-1)e^x + (x-1)^2 e^x =$

$= (2+(x-1))(x-1)e^x = (x+1)(x-1)e^x$

så $f'(x) = 0 \Leftrightarrow x = \pm 1$.

| $x$ | −2 | | −1 | | 1 | | 2 |
|---|---|---|---|---|---|---|---|
| $f'(x)$ | | + | 0 | − | 0 | + | |
| $f(x)$ | $9/e^2$ | ↗ | $4/e$ | ↘ | 0 | ↗ | $e^2$ |




Svar:

$f_{max} = f(2) = e^2$

$f_{min} = f(1) = 0$

5) $\begin{cases} x+y+z=3 \\ x-y+az=5 \\ x+ay-z=1 \end{cases} \Leftrightarrow \begin{cases} x+y+z=3 \\ -2y+(a-1)z=2 \\ (a-1)y-2z=-2 \end{cases}$

$\Leftrightarrow \begin{cases} x+y+z=3 \\ -2y+(a-1)z=2 \\ \frac{a^2-2a-3}{2}z = a-3 \end{cases}$

För $a=-1$: fås

$\begin{cases} x+y+z=3 \\ -2y-2z=2 \\ 0=-4 \end{cases}$ som saknar lösning.