Lösningar

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1) $p(x) = x^3 - 3x^2 - 2x + 4 = 0$

En lösning är $x=1$ så $p(x)$ kan faktoriseras:

$$\begin{array}{r|l} & x^2 - 2x - 4 \\ \hline x-1 & x^3 - 3x^2 - 2x + 4 \\ & -(x^3 - x^2) \\ \hline & -2x^2 - 2x + 4 \\ & -(-2x^2 + 2x) \\ \hline & -4x + 4 \\ & -(-4x + 4) \\ \hline & 0 \end{array}$$

$p(x) = (x-1)(x^2 - 2x - 4)$

$x^2 - 2x - 4 = 0 \iff (x-1)^2 = 5 \iff x = 1 \pm \sqrt{5}$

Svar: $x = 1$ eller $x = 1 \pm \sqrt{5}$

2) $f(x)$ är definierad om $x \neq 0$ och $x - \frac{3}{x} > 0$

$x - \frac{3}{x} > 0 \iff \frac{x^2-3}{x} > 0 \iff \frac{(x+\sqrt{3})(x-\sqrt{3})}{x} > 0$

Teckentabell:

x			$-\sqrt{3}$			0			$\sqrt{3}$		
$x+\sqrt{3}$	$-$	$-$	0	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
$x-\sqrt{3}$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	0	$+$	$+$
x	$-$	$-$	$-$	$-$	$-$	0	$+$	$+$	$+$	$+$	$+$
$\frac{(x+\sqrt{3})(x-\sqrt{3})}{x}$	$-$	$-$	0	$+$	$+$	⧘	$-$	$-$	0	$+$	$+$

så vi får $-\sqrt{3} < x < 0$ eller $x > \sqrt{3}$

Svar: $D_f = \left]-\sqrt{3}, 0\right[\cup \left]\sqrt{3}, \infty\right[$

3) $f(x) = (2+x)e^{-x}$

$f'(x) = 1 \cdot e^{-x} + (2+x)e^{-x}(-1) = -(x+1)e^{-x}$

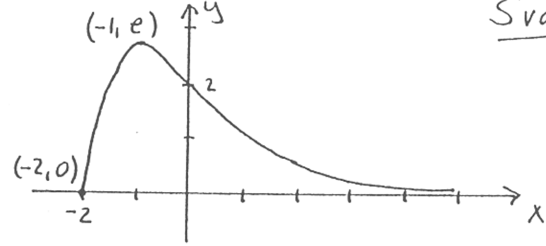
så $f'(x) = 0 \iff x+1 = 0 \iff x = -1$

x	-2			-1			(∞)
$f'(x)$		$+$ $+$ $+$		0		$-$ $-$ $-$	
$f(x)$	0	$\nearrow$		e		$\searrow$	(0)

ty $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{2+x}{e^x} = \text{"}\frac{\infty}{\infty}\text{"} = [\text{l'Hospital}] = \lim_{x\to\infty} \frac{1}{e^x} = 0$

Svar: $f_{min} = f(-2) = 0$

$f_{max} = f(-1) = e$



4) $\lim_{x\to 1} \frac{1 + \overbrace{\cos(\pi x)}^{\to -1}}{1 - \underbrace{x}_{\to 1} + \underbrace{\ln x}_{\to 0}} = \text{"}\frac{0}{0}\text{"} = [\text{l'Hospital}] =$

$= \lim_{x\to 1} \frac{-\pi \overbrace{\sin(\pi x)}^{\to 0}}{-1 + \underbrace{1/x}_{\to 1}} = \text{"}\frac{0}{0}\text{"} = [\text{l'Hospital}] =$

$= \lim_{x\to 1} \frac{-\pi^2 \cos(\pi x)}{-1/x^2} = \frac{-\pi^2 \cos(\pi)}{-1/1^2} = \underline{\underline{-\pi^2}}$

5) a) $\int e^x \sqrt{1+e^x}\, dx = \begin{bmatrix} t = 1 + e^x \\ dt = e^x dx \end{bmatrix} =$

$= \int \sqrt{t}\, dt = \frac{t^{3/2}}{3/2} + C =$

$= \underline{\underline{\frac{2}{3}\left(1+e^x\right)^{3/2} + C}}$