b) $\int_0^1 (x^2+1)e^x\,dx = [P.I.] = \left[(x^2+1)e^x\right]_0^1 - \int_0^1 2x e^x\,dx =$

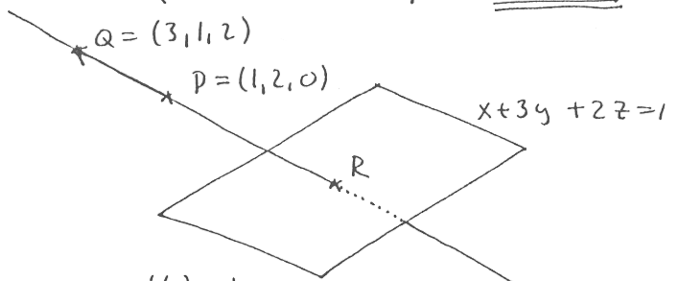
$= (2e^1 - 1\cdot e^0) - 2\int_0^1 x\, e^x\,dx = [P.I.] =$

$= 2e - 1 - 2\left(\left[x e^x\right]_0^1 - \int_0^1 1\cdot e^x\,dx\right) =$

$= 2e - 1 - 2\left((1\cdot e^1 - 0\cdot e^0) - \left[e^x\right]_0^1\right) =$

$= 2e - 1 - 2\left(e - (e-1)\right) = \underline{\underline{2e-3}}$

6)



a) $\vec{PQ}$ är en riktningsvektor för linjen

$\vec{PQ} = (3-1, 1-2, 2-0) = (2,-1,2)$ så

linjen är $\begin{cases} x = 1+2t \\ y = 2-t \\ z = 0+2t \end{cases} \quad t \in \mathbb{R}$

b) Vilket t-värde ger skärningspunkten R ?

$(1+2t) + 3(2-t) + 2(0+2t) = 1$

$\Leftrightarrow$

$3t + 6 = 0$

$\Leftrightarrow$

$t = -2$

vilket insatt i linjens ekvation ger

$\begin{cases} x = 1-4 = -3 \\ y = 2+2 = 4 \\ z = 0-4 = -4 \end{cases}$ $\quad$ Svar: $(-3, 4, -4)$

7) $\underline{x>0}$: $\quad xy' + y = \ln x \Leftrightarrow y' + \frac{2}{x}y = \frac{1}{x}\ln x$

$a(x) = \frac{2}{x}$, $A(x) = 2\ln x$, Integrerande faktor är

$e^{A(x)} = e^{2\ln x} = x^2$ så

$(yx^2)' = x\ln x$

$\int x \ln x\,dx \overset{P.I.}{=} \frac{x^2}{2}\ln x - \int \frac{x^2}{2}\cdot\frac{1}{x}\,dx = \frac{x^2}{2}\ln x - \frac{1}{2}\int x\,dx =$

$= \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$ så

$yx^2 = \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$ dvs $y(x) = \frac{\ln x}{2} - \frac{1}{4} + \frac{C}{x^2}$

$y(1) = 1$ ger $1 = \frac{\ln 1}{2} - \frac{1}{4} + C \Rightarrow C = 5/4$

Svar: $y(x) = \frac{\ln x}{2} - \frac{1}{4} + \frac{5}{4x^2} \quad x>0$.

8) $f(x) = \frac{1}{x^2+1} \qquad f'(x) = -\frac{2x}{(x^2+1)^2}$

Tangenten till $y = f(x)$ i punkten $(a, f(a))$:

$y - f(a) = f'(a)(x-a) \Leftrightarrow$

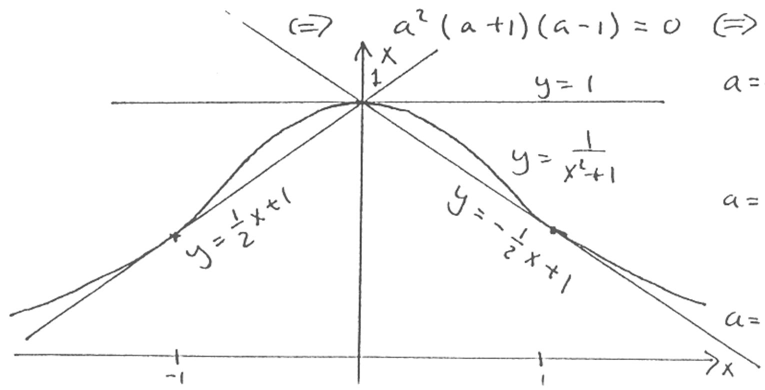
$y - \frac{1}{a^2+1} = -\frac{2a}{(a^2+1)^2}(x-a)$

Den skall gå genom punkten $(x,y) = (0,1)$:

$1 - \frac{1}{a^2+1} = \frac{-2a}{(a^2+1)^2}(0-a) \Leftrightarrow$

$(a^2+1)^2 - (a^2+1) = 2a^2 \Leftrightarrow a^4 - a^2 = 0$

$\Leftrightarrow a^2(a+1)(a-1) = 0 \Leftrightarrow a = -1, 0$ eller 1.



$a=0$ ger $y - 1 = -0(x-0)$

dvs $\underline{\underline{y=1}}$

$a=1$ ger $y - \frac{1}{2} = -\frac{1}{2}(x-1)$

dvs $\underline{\underline{y = -\frac{1}{2}x + 1}}$

$a=-1$ ger $y - \frac{1}{2} = \frac{1}{2}(x+1)$

dvs $y = +\frac{1}{2}x + 1$