MAN460: list of theorems that may appear in exam

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Note that some *exercises* in the exam could be of theory like type, without being in this list below. These can be special cases or variations of the theory we have covered, and could be problems of the type

- *Consider the following problem .... prove that the solutions are positive for all times*

Note that below, unless otherwise stated, *I expect you to be able to state AND prove* the corresponding theorems. Note that I have not given exact references in the books .... if you don't find it in your book, please ask me where to look. All theory that I ask for can be found in my handwritten notes.

1. Formulate and prove a therem on the existence and uniqueness of slutions to first order differential equations, assuming a Lipschitz condition *(note that this is a rather vague statement ... it is part of the problem to give it a mathematically exact form)*
2. prove that the Euler polygon method (other names: the explicit Euler method, the forward Euler method, the Cauchy-Euler method) converges (the theorem is completely stated in the notes)
3. Prove that the set of solutions to a linear, homogeneous differential equation form a linear space. What is the dimension?
4. Properties of the matrix exponential: definition, convergence of power series, proof of the formula $\exp(A+B) = \exp(A)\exp(B)$ if A and B commute.
5. Matrix norms: what are the important properties of a matrix norm; in particular the "operator norm" is important. Estimates of $\exp(At)$ in terms of the eigenvalues of A (this is not explicitly stated, but consider that the solution to a linear equation is given as linear combinations of functions of the form $p(t)e^{\lambda_j t}$, where $p(t)$ are polynomials.
6. Statement of the Cayley-Hamilton theorem, statement of the Jordan form theorem for matrixes.
7. The Gronwall lemma. There are several versions ... the most general form I can ask for is the one in my notes.
8. Construction of a fundamental matrix for an arbitrary constant coefficient linear system. In the notes (and in the book, of course) the fundamental matrix is defined for arbitrary linear systems, and the constant coefficient case is a special case.
9. Definition of stability, asymptotic stability, exponential stability.
10. State a stability theorem and an instability theorem for systmes of the form $\mathbf{y}' = A\mathbf{y} + \mathbf{g}(\mathbf{y}, t)$; you should be able to prove the stability theorem.
11. Lyapunovs stability theorem, the case with stability and asymptotic stability.
12. Definition of Sturm's boundary value problem (and of the corresponding eigenvalue problem).
13. Existence (and definition) of a Green's function to the such boundary value problems
14. Definition of scalar products, of self adjoint operators.